

A MATHEMATICAL MODEL OF THE MOVEMENT OF ROOT CROPS OF A SUGAR BEET AT VIBRATING EXCAVATION

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New mathematical model which describes process of direct withdrawal of a root crop of a ground which is carried out under the action of vertical disturbing forces and traction effort which are given to it by vibrating of digging up working body is constructed. The received systems of the differential equations, which decisions have enabled to find the law of movement of a root crop during its direct vibrating withdrawal.

Introduction

Use of vibrating excavation of a sugar beet root from a ground has a number of essential advantages in comparison with other ways. It is characterized by less damage of root, decrease in losses of a crop at harvesting, more intensive clarification of root crops from the stuck ground, less smaller blocking up of a digger working channel from a ground and residues of weeds. Therefore this technological process requires detailed analytical research, the further development and wide introduction of advanced vibrating digging bodies.

Statement of a problem

Theoretical researches of technological process of vibrating excavation of a sugar beet root from a ground enable scientifically to prove constructive and kinematic parameters of vibrating digging up working bodies. Such investigations are necessary first of all for the theoretical work analysis of vibrating digging up bodies in adverse conditions, on difficult and firm soils where reliability of work of beet-harvesting machines work essentially decreases. In turn the deep theoretical analysis of any technological process is possible only at presence of adequate mathematical models which describe the given process.

The analysis of researches and publications

Thorough theoretical and experimental researches of vibrating excavation of sugar beet root were rather widely described in works [1–6].

So, in work [6] the process of a root crop withdrawal of a ground is considered in most general case – at asymmetrical claw of a root crop by vibrating digging up working body. The given process is described by means of kinematic and dynamic equations of Aler. System of the differential equations received in work [6] describes spatial oscillatory process of the root crop fixed in the ground, as in the spring environment, with one point of fastening. In the given work the process of vibrating withdrawal of a root crop of a ground is considered at symmetric claw of a root crop by both shares of vibrating digging up body.

At such claw of a root crop by digging up shares the process of the further full withdrawal of a root crop of a ground is possible. Therefore we investigate process of direct withdrawal of a root crop of a ground at its symmetric claw by vibrating digging up working body.

The purpose of research

To construct mathematical model of direct withdrawal of a sugar beet root crop from a ground, which is carried out under the action of vertical troubling forces which is given to a root crop from vibrating digging up working body, and traction effort owing to translational movement of digger.

Results of research

At first let's make the necessary formalization of technological process which will be considered. Despite of that process of withdrawal of a ground of a sugar beet a root crop will take place for a short time interval (as translational speed of a root crop machines can reach 2 km/s) all technological process is possible to divide into the separate interconnected, consecutive operations conditionally. As, it was marked above the withdrawal is possible, only at symmetric claw of a root crop by working body and simultaneously with translational fluctuations of a root crop occurs angular fluctuations of a root crop around of a conditional point of fastening on some angle.

At the first stage of withdrawal, and especially at the first fluctuations, renew force at angular fluctuations, and so, and its moment concerning a conditional point of fastening, will be maximal. That is why the angle of inclination of a root crop will be insignificant enough and a full (or partial) restoration of its vertical position owing to translational movement of the digger will be possible. Nevertheless, owing to action of forward translational fluctuations of a root crop together with a ground surrounding it, the compaction of the specified ground will decrease, and renewing force at angular fluctuations will be decrease too. So, with each following fluctuation the angle of an inclination of a root crop will increase, and restoration of the previous position – to decrease. The root crop will be loosened around of a conditional point of fastening with gradual increase of an inclination angle of a root crop forward on a digger course. It will lead to the break of a root crop connections with a ground in the direction of digger's movement, beginning from the top part of a conic surface of a root crop in unloosened ground, gradually approaching to a conditional point of its fastening. So as it was stated above it follows, that the destruction of connections of a root crop with a ground occurs simultaneously in two directions – along the translational digger's movement and in the direction perpendicularly to specified (on depth of a root crop arrangement in the ground). Thus the force of connections of a root crop with a ground and the forces of a ground spring will gradually decrease to such minimal size when oscillatory processes will pass into the processes of continuous moving of a root crop upwards and forward – along the translational digger movement, and also continuous turn of a root crop turn around of its mass center on some angle to full withdrawal of a root crop from a ground. Forces of a ground spring will simply pass into the force of resistance of the loosened ground at a root crop movement in a digger working channel. After that the stage of direct withdrawal of a sugar beet root crop from a ground is begun.

For construction of mathematical model first of all we shall make the equivalent scheme of a root crop interaction with working surfaces of vibrating digging up working body at its direct withdrawal (fig.). For this purpose we shall present a vibrating digging up working body in the form of two coupled digging up surfaces (wedges) $A_1B_1C_1$ and $A_2B_2C_2$, each of which in spacious has an inclination under angles α , β , γ and which are established thus one to one, that the working channel is formed, the back part of it is narrowed. The specified wedges carry out oscillatory movements in longitudinal-vertical planes (the mechanism of shares drive into oscillatory

movement it is not shown), with corresponding amplitude and frequency. A direction of translational movement of vibrating digging up working body is shown by an arrow. Projections of points B_1 and B_2 to an axis O_1y_1 are designated by points D_1 and D_2 accordingly.

We consider, that with surfaces of wedges $A_1B_1C_1$ and $A_2B_2C_2$. In the corresponding points the root crop which is approximated by a body of the cone-shaped form interacts, and the claw of a root crop by a working body occurs symmetrically from its both sides.

Let's assume further, that the working surface of a wedge $A_1B_1C_1$ carries out a direct contact with a root crop in a point K_1 , and a surface $A_2B_2C_2$ – in a point K_2 . The straight lines lead through points of a root crop contact K_1 and K_2 and points B_1 and B_2 form on section with the sides of wedges A_1C_1 and A_2C_2 corresponding points M_1 and M_2 . Thus, δ is dihedral angle ($\angle B_1M_1D_1$) between the bottom basis $A_1D_1C_1$ and a working surface of a wedge $A_1B_1C_1$ or accordingly a dihedral angle ($\angle B_2M_2D_2$) between the bottom basis $A_2D_2C_2$ and a working surface of a wedge $A_2B_2C_2$.

Let's show forces which arise owing to interaction of a root crop with vibrating working body.

Let from vibrating digging up working body the vertical troubling \bar{Q}_{tr} , force operates which changes under the harmonious law of such kind:

$$Q_{tr} = H \sin \omega t. \quad (1)$$

Where H – amplitude of troubling forces; ω – frequency of troubling forces.

The given force play the basic role during loosening a ground in a zone of a digger working channel and withdrawal of a root crop. A specified troubling force \bar{Q}_{tr} is applied to a root crop from its two sides and on the scheme it is presented by two compounds $\bar{Q}_{tr.1}$ and $\bar{Q}_{tr.2}$. The given forces are applied accordingly in points K_1 and K_2 on distance h from a conditional point of fastening O and they cause fluctuation of a root crop in longitudinal-vertical planes which destroy connections of a root crop with a ground and create for it a condition of withdrawal from a ground.

As a claw of a root crop is symmetric it is obvious, that there will be a following correlation:

$$Q_{tr.1} = Q_{tr.2} = \frac{1}{2} H \sin \omega t. \quad (2)$$

Let's disintegrate the given forces into normal \bar{N}_1 and \bar{N}_2 and tangents compounds \bar{T}_1 and \bar{T}_2 , as it is shown on fig. As vibrating digger moves translationally in a direction of an axis O_1x_1 According to a root crop which is fixed in a ground and during the moment of claw of a root crop by the working body in the direction of an axis O_1x_1 Motive forces operate also \bar{P}_1 and \bar{P}_2 . Also Let's disintegrate the forces \bar{P}_1 and \bar{P}_2 in two compounds: normal \bar{L}_1 and \bar{L}_2 and tangents \bar{S}_1 and \bar{S}_2 to surfaces $A_1B_1C_1$ and $A_2B_2C_2$ accordingly.

Besides in points of contact K_1 and K_2 forces of friction act \bar{F}_{K1} and \bar{F}_{K2} accordingly

which counteract to a root crop sliding on a working surface of wedges $A_1B_1C_1$ and $A_2B_2C_2$ during its claw by vibrating working body also K_2 operate. Vectors of these forces are directed opposite to a vector of relative speed of a root crop sliding on a surface of wedges.

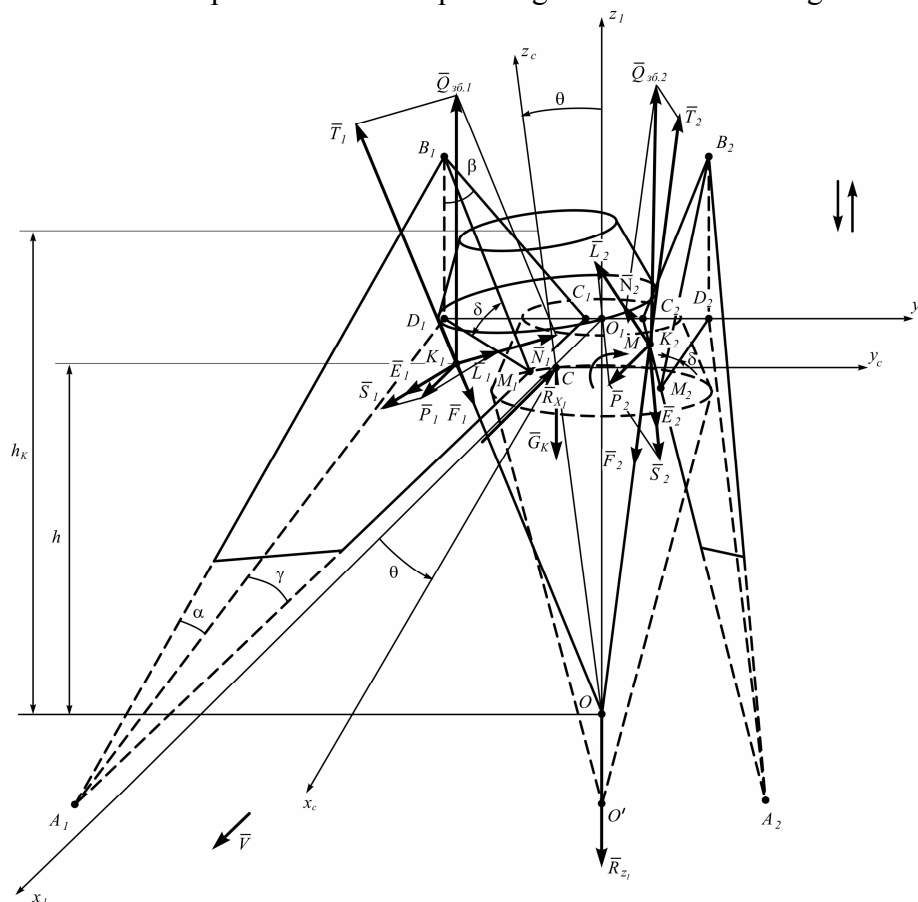


Fig. - the Equivalent scheme of vibrating excavation of a sugar beet root crop from a ground

A root crop sliding on a surface of wedges can occur in a direction of forces action \bar{T}_1 , \bar{T}_2 (in parallel lines B_1M_1 and B_2M_2) in the direction, opposite to the actions of forces \bar{S}_1 , \bar{S}_2 , due to forces of resistance of a ground.

The vector of relative speed of a root crop sliding on a surface of wedges can be disintegrated into the compounds in the directions specified above. So, force of friction \bar{F}_{K1} also can be disintegrated into compounds: \bar{F}_1 in a direction, opposite to vector \bar{T}_1 , and \bar{E}_1 – in a direction of a vector \bar{S}_1 .

Similarly, force of friction \bar{F}_{K2} also can be disintegrated on two compounds: \bar{F}_2 – in a direction, opposite to vector \bar{T}_2 , and \bar{E}_2 – in a direction of a vector \bar{S}_2 .

It is obvious, that $F_1 = F_2$, $E_1 = E_2$.

In the center of a root crop mass (a point C) force of a root crop mass operates \bar{G}_k . Forces of resistance of the loosened ground at movement of a root crop in a working channel of digger in a direction of axes O_{1x1} and O_{1z1} are designated through \bar{R}_{x1} and \bar{R}_{z1} accordingly.

At direct withdrawal of a root crop from a ground the turn of a root crop around of its center of mass (point C) will be carried out Under the action of pair of resistance forces of the loosened ground. We shall designate the moment of this pair of forces as M .

At direct withdrawal of a root crop it is possible to consider forces of resistance of the loosened ground dependent on speed of a root crop movement in the loosened ground or as a first approximation – simply constants. Therefore for simplification of mathematical model we shall consider the forces \bar{R}_{x1} , \bar{R}_{z1} and the moment of pair M as constants.

Let's make at first the differential equations of movement of the center of a root crop mass (point C), i.e. translational movement of a root crop along axes O_{1x1} and O_{1z1} . Considering the given above scheme of forces, the differential equation of movement of the mass centre of a root crop in the vector form at its direct withdrawal will be the following:

$$m_k \bar{a} = \bar{N}_1 + \bar{N}_2 + \bar{L}_1 + \bar{L}_2 + \bar{F}_1 + \bar{F}_2 + \bar{E}_1 + \bar{E}_2 + \bar{G}_k + \bar{R}_{z1} + \bar{R}_{x1}, \quad (3)$$

Where \bar{a} – acceleration of movement of the mass center of a root crop.

As the process of withdrawal as it has been specified above, occurs at symmetric claw of a root crop by working body, so the movement of a root crop along a working channel of the digger occurs actually in longitudinal-vertical planes (planes $x_1 O_{1z1}$) that is why the vector equation (3) is reduced to system of two equations in projections to axes O_{x1} and O_{z1} .

After the definition of values of all forces which enter into the vector equation (3), and their projections to axes O_{x1} and O_{z1} we shall receive two following systems of the differential equations:

$$\left. \begin{aligned} \ddot{x} &= \frac{1}{m_k} \left[\frac{\cos \delta \operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} + f \cos^2 \delta \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \sin \gamma + \right. \\ &+ \left. f \cos \delta \cos \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \cos \gamma \right] H \sin \omega t + \frac{2}{m_k} \times \\ &\times \left[\frac{\sin \gamma \operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} + f \sin^2 \gamma \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \cos \delta + \right. \\ &+ \left. f \sin \gamma \cos \gamma \cos \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \right] P_1 - \frac{R_{x1}}{m_k}, \\ \ddot{z} &= \frac{1}{m_k} \left[\frac{\cos \delta \operatorname{tg} \beta}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} - f \cos \delta \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \sin \delta \right] H \sin \omega t + \\ &+ \frac{2}{m_k} \left[\frac{\sin \gamma \operatorname{tg} \beta}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} - f \sin \gamma \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \sin \delta \right] P_1 - \frac{R_{z1}}{m_k} - g, \\ \omega t &\in [2k\pi, 2(k+1)\pi], \quad k = 0, 1, 2, \dots \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} m_k \ddot{x} &= \frac{2P_1 \sin \gamma \operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} + 2fP_1 \sin^3 \gamma \cos \delta + fP_1 \sin 2\gamma \cos \gamma - R_{x1}, \\ m_k \ddot{z} &= \frac{2P_1 \sin \gamma \operatorname{tg} \beta}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} - 2fP_1 \sin^2 \gamma \sin \delta - G_k - R_{z1}, \\ \omega t &\in [(2k-1)\pi, 2k\pi], \quad k=1, 2, \dots \end{aligned} \right\} \quad (5)$$

Thus the system of the differential equations (4) describes the process of directly vibrating withdrawal of a sugar beet root crop of a ground (i.e. a piece on which a periodic troubling force acts on a root crop), and the system of the differential equations (5) describes the process of withdrawal of a root crop of a ground when it is not acted by a troubling force. i.e. the same vibrating of digging up the working body in the different time intervals can carry out the process of excavation of a root crop as usual share digger.

Let's solve the received systems of the differential equations.

For the given systems of the differential equations (4), (5) initial conditions will be the following:

$$\begin{aligned} \text{At } t = 0: \quad \dot{x} &= 0, & \dot{z} &= 0, & (6) \\ x_1 &= x_{10}, & z_1 &= -\frac{1}{3}h_k. & (7) \end{aligned}$$

The system of the differential equations (4) is the system of the linear differential equations of the second order. As it is known, it is solved in quadratures. For the simplification of the record system of the differential equations (4) we shall introduce the following designations:

$$\begin{aligned} \frac{1}{m_k} \left[\frac{\cos \delta \operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} + f \cos^2 \delta \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \sin \gamma + \right. \\ \left. + f \cos \delta \cos \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \cos \gamma \right] = \phi_1, \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{2}{m_k} \left[\frac{\sin \gamma \operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} + f \sin^2 \gamma \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \cos \delta + \right. \\ \left. + f \sin \gamma \cos \gamma \cos \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \right] = \psi_1, \end{aligned} \quad (9)$$

$$\frac{1}{m_k} \left[\frac{\cos \delta \operatorname{tg} \beta}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} - f \cos \delta \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \sin \delta \right] = \phi_2, \quad (10)$$

$$\frac{2}{m_k} \left[\frac{\sin \gamma \operatorname{tg} \beta}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} - f \sin \gamma \sin \left(\gamma + \frac{\alpha_{K1 \max}}{2} \right) \sin \delta \right] = \psi_2. \quad (11)$$

Considering expressions (8) – (11), the system of the differential equations (4) will get a following kind:

$$\left. \begin{aligned} \ddot{x} &= \phi_1 H \sin \omega t + \psi_1 P_1 - \frac{R_{x1}}{m_k}, \\ \ddot{z} &= \phi_2 H \sin \omega t + \psi_2 P_1 - \frac{R_{z1}}{m_k} - g. \end{aligned} \right\} \quad (12)$$

We will integrate the system of the differential equations (12). After twofold integration and a finding of any arbitrary constants we receive the following solutions of the differential equations (4) in a final kind:

$$\left. \begin{aligned} \dot{x}_1 &= -\frac{\phi_1 H}{\omega} \cos \omega t + \psi_1 P_1 t - \frac{R_{x_1} t}{m_k} + \frac{\phi_1 H}{\omega}, \\ \dot{z}_1 &= -\frac{\phi_2 H}{\omega} \cos \omega t + \psi_2 P_1 t - \frac{R_{z_1} t}{m_k} - g t + \frac{\phi_2 H}{\omega}. \end{aligned} \right\} \quad (13)$$

$$\left. \begin{aligned} x_1 &= -\frac{\phi_1 H}{\omega^2} \sin \omega t + \frac{\psi_1 P_1 t^2}{2} - \frac{R_{x_1} t^2}{2m_k} + \frac{\phi_1 H t}{\omega} + x_{10}, \\ z_1 &= -\frac{\phi_2 H}{\omega^2} \sin \omega t + \frac{\psi_2 P_1 t^2}{2} - \frac{R_{z_1} t^2}{2m_k} - \frac{g t^2}{2} + \frac{\phi_2 H t}{\omega} - \frac{1}{3} h_k. \end{aligned} \right\} \quad (14)$$

Systems of the equations (13) and (14) describe the laws of the speed change and the moving of the mass centre of a root crop during its direct withdrawal from a ground. From the second equation of system (14) it is possible to define the time t of the direct withdrawal of a root crop from a ground. For this purpose it is necessary to substitute in the left part of the specified equation the value $z_1 = 0$ and to solve the received equation according to t . As the equation is transcendental to receive analytical expression for definition t is impossible, nevertheless it can be solved on PC by means of known numerical methods. The calculated mean t_1 can be applied to the definition of the unit productivity for a root crop. Excavation by vibrating digging up working bodies.

Let's solve the system of the differential equations (5). For the simplification of the given record system we shall also take the following designations:

$$\frac{1}{m_k} \left(\frac{2 \sin \gamma \operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} + 2f \sin^3 \gamma \cos \delta + f \sin 2\gamma \cos \gamma \right) = \psi'_1, \quad (15)$$

$$\frac{1}{m_k} \left(\frac{2 \sin \gamma \operatorname{tg} \beta}{\sqrt{\operatorname{tg}^2 \gamma + 1 + \operatorname{tg}^2 \beta}} - 2f \sin^2 \gamma \sin \delta \right) = \psi'_2. \quad (16)$$

In view of expressions (15), (16) the system of the differential equations (5) will get a kind:

$$\left. \begin{aligned} \dot{x}_1 &= \psi'_1 P_1 - \frac{R_{x_1}}{m_k}, \\ \dot{z}_1 &= \psi'_2 P_1 - \frac{G_k}{m_k} - \frac{R_{z_1}}{m_k}, \end{aligned} \right\} \quad (17)$$

$$\omega t \in [(2k-1)\pi, 2k\pi], \quad k = 1, 2, \dots$$

After twofold integration of system of the equations (17) and a finding of any constants we shall receive denouement systems of the differential equations (5) in a final kind:

$$\left. \begin{aligned} x_1 &= \psi'_1 P_1 t - \frac{R_{x_1}}{m_k} t, \\ z_1 &= \psi'_2 P_1 t - \frac{G_k}{m_k} t - \frac{R_{z_1}}{m_k} t, \end{aligned} \right\} \quad (18)$$

$$\omega t \in [(2k-1)\pi, 2k\pi], \quad k = 1, 2, \dots$$

$$\left. \begin{aligned} x_1 &= \psi'_1 P_1 \frac{t^2}{2} - \frac{R_{x_1} t^2}{2m_k} + x_{10}, \\ z_1 &= \psi'_2 P_1 \frac{t^2}{2} - \frac{G_k t^2}{2m_k} - \frac{R_{z_1} t^2}{2m_k} - \frac{1}{3} h_k, \\ \omega t &\in [(2k-1)\pi, 2k\pi], \quad k=1, 2, \dots \end{aligned} \right\} \quad (19)$$

Systems of the equations (18) and (19) accordingly describe the laws of the speed change and moving of the mass center of a root crop mass during its direct withdrawal out of a ground at the absence of troubling forces action.

Let's work out the differential equation of turn of a root crop around of its center of mass, or around of a conditional axis Cy_c which passes through the center of mass (point C) in parallel axis O_1y_1 . According to [8], the specified equation in a general view will have such view:

$$I_{y_c} \frac{d^2\theta}{dt^2} = M_{y_c}^e, \quad (20)$$

Where θ – an angle of turn of a root crop around axis Cy_c ; I_{y_c} – the moment of inertia of a root crop concerning an axis Cy_c ; $M_{y_c}^e$ – the rotary moment around of an axis Cy_c (the sum of the moments of all external forces which act on a root crop, concerning an axis Cy_c).

The moment of inertia I_{y_c} of a root crop concerning an axis Cy_c is defined according to [8] of such expression:

$$I_{y_c} = \left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon \right) m_k h_k^2. \quad (21)$$

Substituting expressions (2), (21) in the differential equation (20) and carrying out the necessary transformations we shall receive the differential equation of turn of a root crop around axis Cy_c at direct vibrating withdrawal from a ground (i.e. at the action of troubling forces on it) which has a following view:

$$\begin{aligned} \left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon \right) m_k h_k^2 \frac{d^2\theta}{dt^2} &= -H(-h_k + h - z_1) \sin\theta \sin\omega t + 2P_1 \cos\theta(-h_k + h - z_1) + \\ &+ 2 \left(\frac{1}{2} f H \cos\delta \sin\omega t + f P_1 \sin\gamma \right) \sin(\gamma + \alpha_{K1 \max} \sin\omega t) \cos\varepsilon (-h_k + h - z_1) \sin\theta + \\ &+ 2 \left(\frac{1}{2} f H \cos\delta \sin\omega t + f P_1 \sin\gamma \right) \cos(\gamma + \alpha_{K1 \max} \sin\omega t) \cos\gamma (-h_k + h - z_1) \cos\theta - \\ &- M, \\ \omega t &\in [2k\pi, (2k+1)\pi], \quad k=0, 1, 2, \dots \end{aligned} \quad (22)$$

The differential equation of turn of a root crop around axis Cy_c at usual withdrawal (i.e. at the absence of troubling forces), has a following view:

$$\begin{aligned} \left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon \right) m_k h_k^2 \frac{d^2\theta}{dt^2} &= 2P_1 \cos\theta(-h_k + h - z_1) + 2f P_1 \sin^2 \gamma \times \\ &\times \cos\varepsilon (-h_k + h - z_1) \sin\theta + f P_1 \sin 2\gamma \cos\gamma (-h_k + h - z_1) \cos\theta - M, \\ \omega t &\in [(2k-1)\pi, 2k\pi], \quad k=1, 2, \dots \end{aligned} \quad (23)$$

Let's analyze the received differential equations (22) and (23). The differential equation (22) is nonlinear. It is possible to solve it by the approached numerical methods with application of PC, and for each step of application of numerical algorithm of mean z_1 is necessary to find from the second equation of system (14) for the corresponding moment of time t_k . The differential equation (23) which includes the magnitude z_1 which is replaceable and, is also nonlinear, and for each moment of time t_k the mentioned magnitude z_1 is necessary for defining from the second equation of system (19).

Thus, it is finally possible to consider, that the constructed mathematical model of direct withdrawal process of a sugar beet root crop from a ground at its vibrating excavation. The received results enable to define kinematic modes of vibrating excavation root crops at the conditions of inviolate and constructive parameters of vibrating digging up bodies.

Conclusions

1. Two systems of the differential equations which describe plane-parallel movement of a root crop in a ground at its direct withdrawal which is carried out under action of vertical troubling forces which is given to a root crop from vibrating digging up body, and traction effort which arises owing to translational movement of digger.

2. Solution the given differential equations gives an opportunity to find out the law of movement of a root crop in longitudinal-vertical plane at direct withdrawal from a ground.

3. The received results enable also to define kinematic modes of vibrating excavation of root crops from their conditions of inviolate not damage and to find rational constructive parameters of vibrating digging up working bodies.

The list of the literature

1. *Vasylenco P.M., Pogorelyj L.V., Bray V.V. Vibrating way of root crops harvesting // Mechanization and electrification of a socialist agriculture, 1970, №2. – p. 9 – 13.*
2. *Beet-harvesting machines (designing and calculation) // L.V. Pogorelyj, N.V. Tatyanko, V.V. Bray, etc.: under gener. red. Pogorelyj L.V. – Kiev: Technics, 1983. – 168 p.*
3. *Bulgakov V.M., Golovach I.V., Vojtyuk D.G. The Theory of vibrating excavation of the root crops – Collection of scientific works of National agrarian university “Mechanization of an agricultural production”. – 2003, Volume XIV. – p. 34 – 86.*
4. *Bulgakov V.M., Golovach I.V., Vojtyuk D.G. The Theory of cross-section fluctuations of a root crop at vibrating excavation. - works of Tavriysky state agrotechnical academy. Issue 18. Melitopol, 2004. – p. 8 – 24.*
5. *Bulgakov V.M., Golovach I.V. About the compelled cross-section fluctuations of a root crop at vibrating excavation. – Bulletin of the Kharkov national technical university of an agriculture of Peter Vasilenko: Collection of scientific works. Issue 39. Kharkov, 2005. – p. 23 - 39.*
6. *Bulgakov V.M., Golovach I.V. Development of mathematical model of withdrawal of a root crop from a ground // Technics of agrarian and industrial complex, 2006, №6-7. – p. 36-38; №8. – p. 25-28; №9-10. – p. 47-49.*
7. *Bulgakov V.M., Golovach I.V. Specified theory of digging up working body of share type // Bulletin of an agrarian science of Black Sea Coast. Special issue 4 (18). Volume – Nikolaev, 2002. – p. 37-63.*
8. *Butenin N.V., Lunts I.L., Merkin D.R. A rate of theoretical mechanics. Volume II. Dynamics. – M.: Science, 1985. 496 p.*