О.М. Джеджула, О.П. Миколюк,

С.І. Кормановський

Descriptive Geometry

НАРИСНА ГЕОМЕТРІЯ

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Рецензенти:

І.П.Паламарчук, доктор технічних наук, професор

В.Ю.Кучерук, доктор технічних наук, професор

В.І.Клочко, доктор педагогічних наук, професор

Джеджула О.М., Миколюк О.П., Кормановський С.І.

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В посібнику розглянуті основні теоретичні положення курсу «Нарисна геометрія», викладені методи побудови зображень геометричних образів на площині та графічні ілюстрації метричних та позиційних задач. Посібник підготовлено для студентів напрямів інженерії: «Машинобудування», «Процеси, машини та обладнання агропромислового виробництва».

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Introduction

Descriptive geometry belongs to the disciplines, which form the engineering training of specialists of higher technical education.

The aim of the descriptive geometry course is to give the students knowledge and skills to display spatial forms on a plane and to imagine an object form according to its plane image.

The subject of descriptive geometry is the variety of geometric images and the correlation between them.

The tasks of descriptive geometry are the following:

1) to study the theoretical fundamentals of image construction of points, straight lines, planes, surfaces;

2) to solve the problems of mutual belonging and mutual intersection of a straight line and a plane, of two planes, of a straight line and a surface, of a plane and a surface, of two surfaces;

3) to study the methods of drawing conversion;

4) to form spatial, abstract, logical thinking of students

The form constructing elements of space are geometric images like a point, a straight line and a plane which make up more complex figures.

There are some symbols used in this textbook, e.g.:

A, B, C, D, ..., 1, 2, 3, 4… – points;

a, b, … l, m, n … – straight lines and curves;

h – a horizontal straight line;

f – a front straight line;

p – a profile straight line;

α, β, γ, … – planes;

α, β, γ, … – angles;

$\mathbf{\Pi_1}$ – a horizontal projection plane;

$\mathbf{\Pi_2}$ – a front projection plane;

$\mathbf{\Pi_3}$ – a profile projection plane;

α – point A belongs to plane α;

x, y, z – projection axes

Unit 1. A PROJECTION METHOD

Image construction in descriptive geometry is based on a projection method. Let us examine projection elements.

Fig.1 shows points A and S, located in space above plane Π_1. We shall call plane Π_1 a horizontal projection plane. Let us draw a straight line through points A and S to intersect plane Π_1, which is called a horizontal plane. Line SA_1 is called a projecting ray, point A is a projection object, A_1 is a projection of point A onto plane Π_1, S is a projection centre.

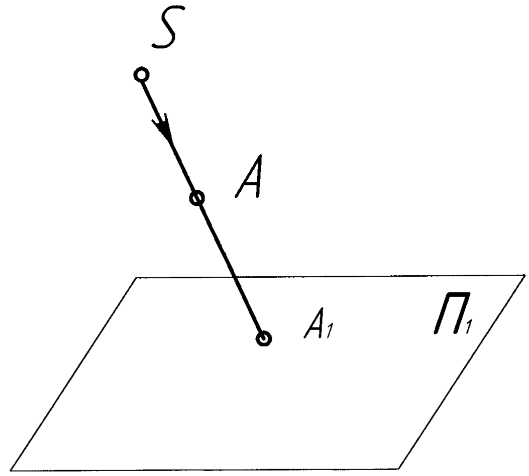


Fig. 1

A projection can be central and parallel.

A central projection is a projection, due to which all projecting straight lines exit from one point (Fig.2). Centre S is proper in this case. If a projection centre is infinitely moved afar, the projecting rays become parallel to each other. Such projection is called a parallel projection (fig.3).

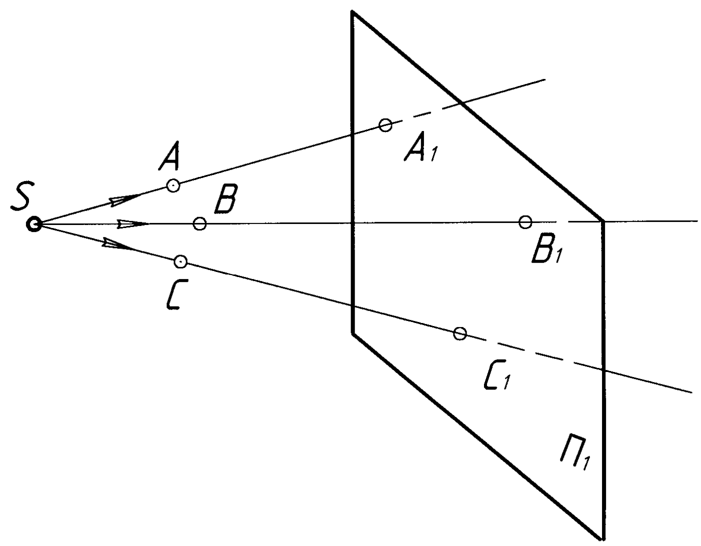


Fig. 2

Centre S is improper by a parallel projection.

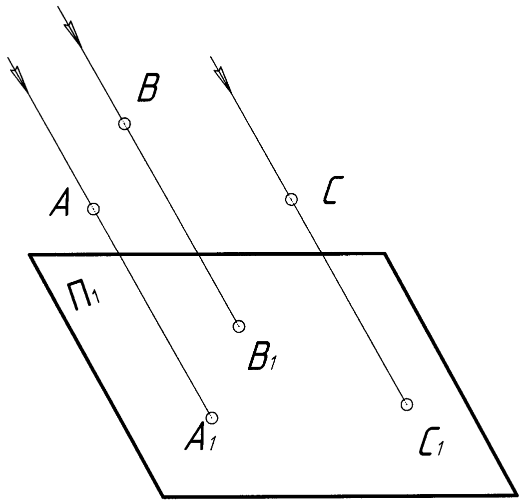


Fig. 3

The rays of a parallel projection and a projection plane form oblique or right angles. Thus, oblique-angled and right-angled (orthogonal) projections are distinguished (fig.4).

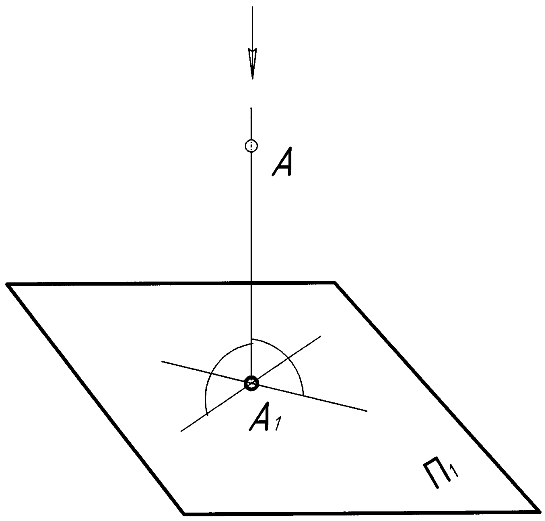


Fig.4

According to a projection method projections can be called central or parallel ones. Parallel projections in their turn are divided into oblique-angled or right-angled ones.

The most common are used to be right-angled projections, as they preserve to a great extent the actual sizes of objects and their elements, besides they have a simpler construction.

1.1. Orthogonal projection properties

Descriptive geometry course studies problems that can be divided into two types: positional ones (problems of mutual position of geometric objects) and metric ones (problems of measuring natural sizes of segments, angles, plane figures, etc.).

According to these problems types orthogonal projection properties can be divided into metric and positional ones.

We will study projection properties successively, examining projections of various geometric elements on projection planes.

Questions to unit "A projection method"

1. What is a projection method?
2. Which projection is called a central projection?
3. Which projection is called a parallel projection?
4. What is a projecting ray?
5. Why are right-angled projections are the most common ones?

Unit 2. A POINT

For construction of a projection of point A (fig.5) onto a horizontal projection plane Π_1 we shall draw through this point a projecting ray to intersect plane Π_1. Point A_1 is a projection of point A. Any number of points marked on a projecting ray will be projected into one point A_1. Thus, one point projection doesn't determine its position in space, because this projection is the projection of any point, which belongs to a projecting ray.

To determine a point position in space you need to have at least two of its projections.

A point position in space will be determined if one constructs point projections onto two projection planes, which are located at a right angle to each other (Π_1 is a horizontal projection plane, Π_2 is a front projection plane).

Let's examine projections of point A onto two mutually perpendicular projection planes (fig.6).

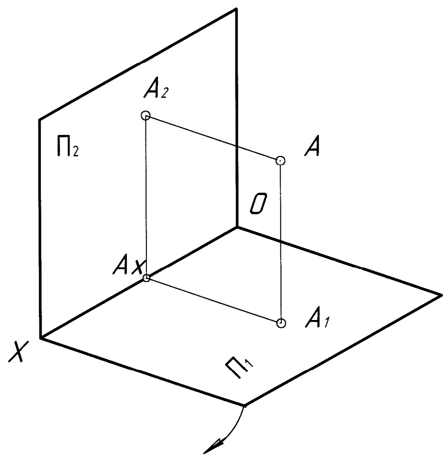


Fig.5

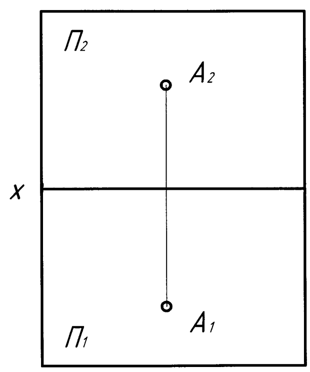


Fig.6

To construct a projection of point A onto plane Π_1 we put down a perpendicular from this point onto plane Π_1. Similarly we construct a front projection of point A onto plane Π_2 – it will be point A_2.

Then we rotate plane Π_1 in relation to axis X_{12} to a superposition with plane Π_2 and we get a plane drawing – a diagram (fig.7).

Axis OX_{12} is a projection axis. So, points A_1, A_2 are a horizontal and a front projections of point A.

Straight line $A_1 – A_2$ is called a link line.

As projecting rays AA_1 and AA_2 are perpendicular to a projection plane, the link line is perpendicular to a projection axis.

Rectangle $AA_2A_{12}A_1$ sides $AA_2 = A_1A_{12}$ and $AA_1 = A_2A_{12}$. Thus, the distance between point A and horizontal projection plane Π_1 is determined by segment A_2A_{12}, and the distance between point A and plane Π_2 is determined by segment A_1A_{12}.

One can get various graphic projection systems depending on projection plane positions and projection centres. The most common system is a rectangular projection system or Monge's method.

A combination of several figure projections (at least two of them) connected with each other is called a rectangular (orthogonal) projection system or Monge's complex graphic.

2.1. A point projection onto three projection planes

Let's examine a point, which lies in the system of three mutually perpendicular projection planes (fig.7). Projection planes Π_1 and Π_2 and also a construction of projections of points A – A_1 and A_2 are already known.

Planes Π_1, Π_2, Π_3 on fig.7 made up the first quarter of space or the first quadrant. Coordinates in the first quarter of space have positive values.

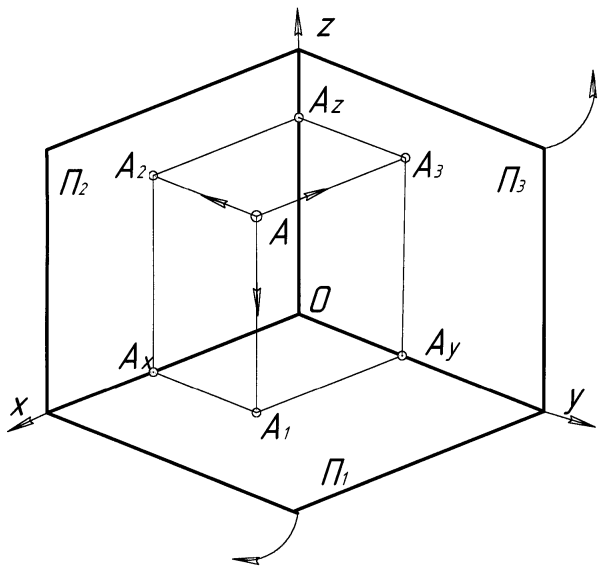


Fig.7

Projection plane Π_3 is called a profile projection plane. If a perpendicular is put down from point A to intersect Π_3, a profile projection of point A – A_3 is got. Projection axes are marked as OX_{12}, OY_{12}, OZ_{23}.

A point position in space is specified by its coordinates. Coordinates of point A are Xa, Ya, Za (i.e. abscissa, ordinate, applicant).

A distance between point A and plane Π_1 is determined by coordinate Za, between point A and plane Π_2 – by coordinate Ya, between point A and plane Π_3 – by coordinate Xa.

To make up a diagram (fig.8), plane Π_1 should rotate to be superposed with plane Π_2, and then plane Π_3 should rotate to be superposed with plane Π_2. Front projection plane Π_2 remains in its place. Axes X_{12} and Z_{23} will not change their position (as they are related to Π_2), and axis OY_{13} will have two directions. Two adjacent point projections will lie on one link line.

To construct a horizontal point projection according to its coordinates, it is necessary to know coordinates Xa; Ya. A front projection of point A which is in the first quarter lies above axis X_{12}, and a horizontal one – below the axis.

The construction of a front point projection is done by coordinates Xa and Za, of a profile one – by coordinates Ya and Za.

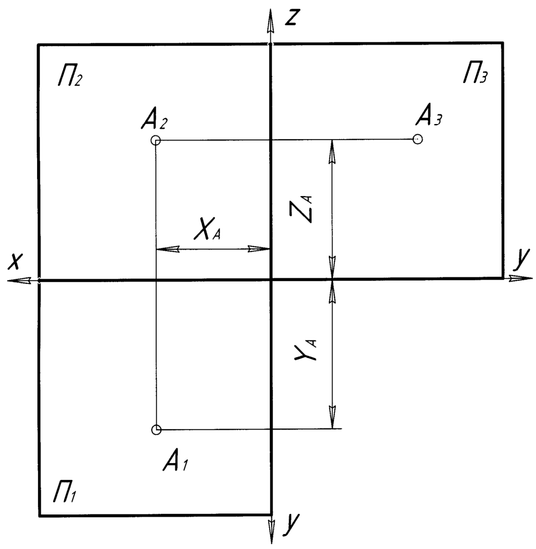


Fig.8

If one of the point coordinates is equal to zero, then the point belongs to one of the projection planes. In this case two of the point projections lie on the axes. For example, fig.9 shows projections of point A which belongs to plane Π_2 (i.e. coordinate Y is equal to zero); fig.10 shows projections of point A that belongs to plane Π_3 (coordinate X is equal to zero). For the point that belongs to plane Π_1 coordinate Z is equal to zero.

If two point coordinates are equal to zero, the point belongs to a projection axis.

For example, point A lies on axis X (fig.11). In this case two of its coordinates Y and Z are equal to zero. One of the projections (A_3) coincides with origin.

Fig. 12 shows point A which lies on axis Y. For such a point position in space coordinates X and Z are equal to zero. A front projection of point A_2 coincides with origin.

For the point which lies on axis Z there will be its profile projection in the coordinates centre.

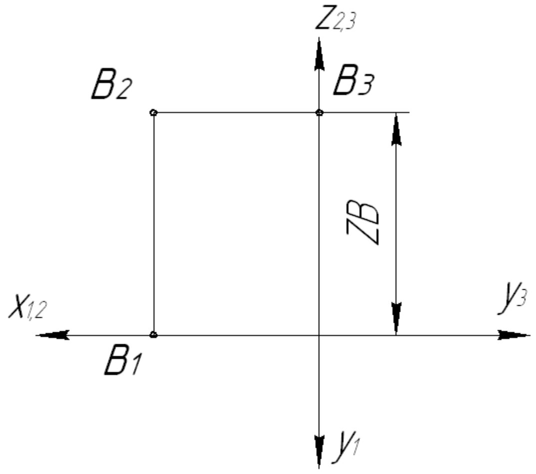


Fig.9

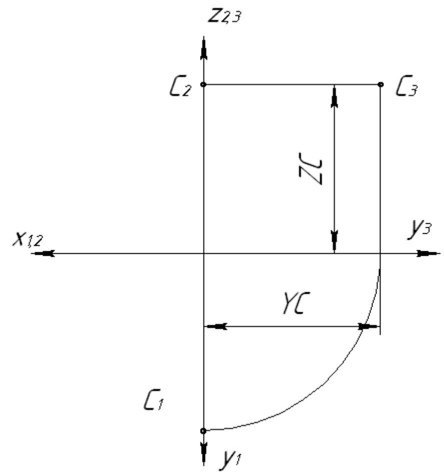


Fig.10

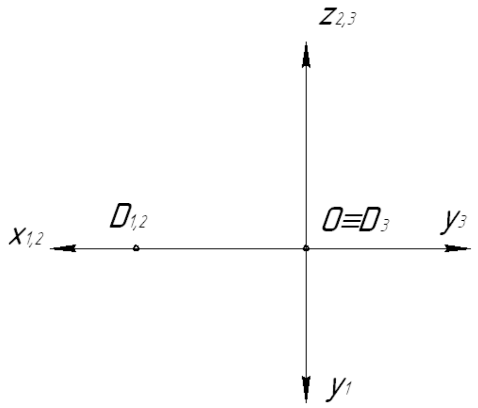


Fig.11

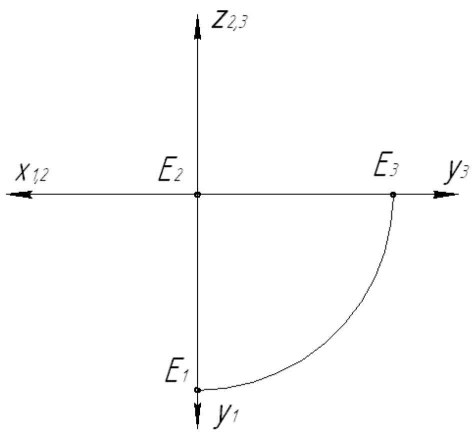


Fig.12

2.2. A point in different space quarters

Space is divided into four quarters (or quadrants) by projection planes Π_1 and Π_2 (fig.13).

To get a diagram we rotate projection plane Π_1 in relation to axis OX_{12} clockwise to superposition with plane Π_2. Herewith, front half-plane Π_1 coincides with lower half-plane Π_2, and a back half-plane coincides with an upper half-plane. The axes location is shown on fig.14.

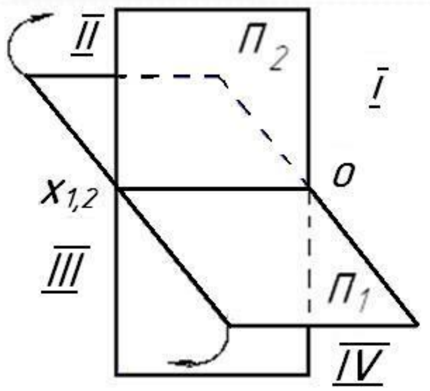


Fig.13

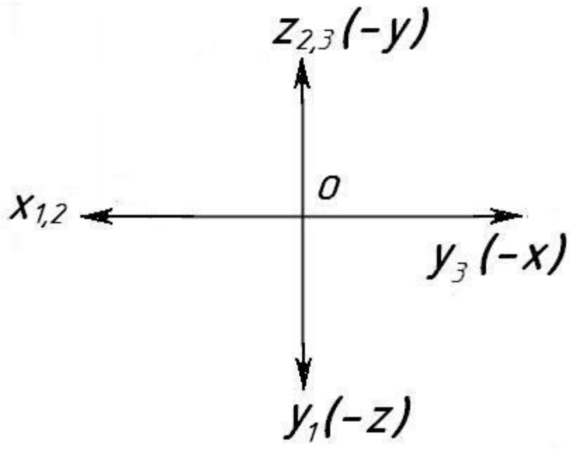


Fig.14

If a point lies in the first quarter, its front projection will be placed above axis OX_{12} on the diagram, and its horizontal projection will be located under axis OX_{12} (fig.15, 16).

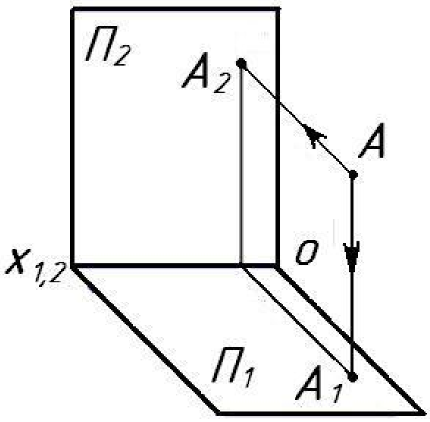


Fig.15

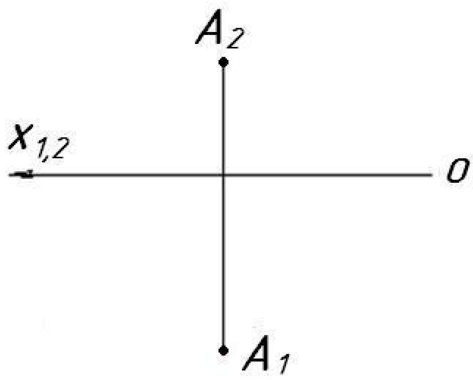


Fig.16

If a point lies in the second quarter, its projections will be located above axis OX_{12} on the diagram (fig.17, 18).

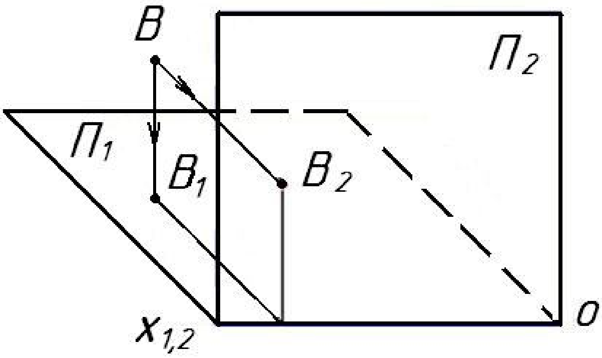


Fig.17

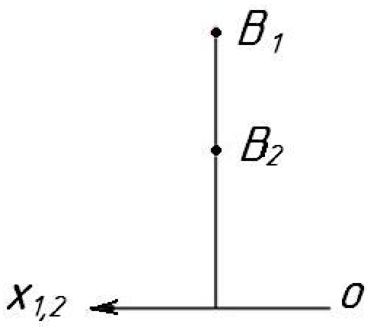


Fig.18

If a point lies in the third quarter, its horizontal projection will be placed above axis OX_{12} on the diagram, and its front projection – under it (fig.19, 20).

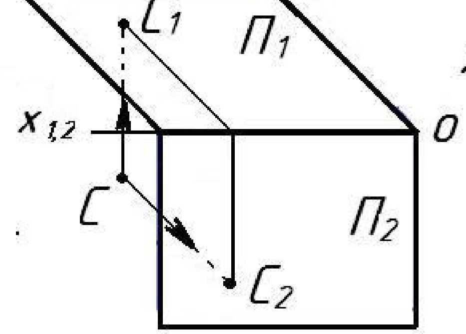


Fig.19

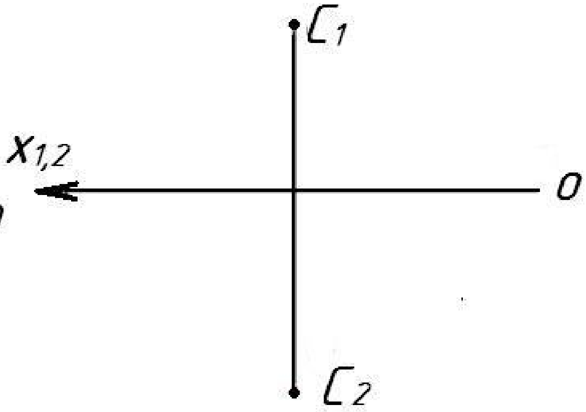


Fig.20

If a point lies in the fourth quarter, its horizontal and front projections will be located under axis X_{12} (fig.21, 22).

Till now Monge's complex graphic was done due the availability of axis X_{12}, which divided projection fields. But the availability of this axis is not always necessary, the projection view doesn't depend on it. The most common graphic in technical drawing is a graphic without axis, when axis is absent. The graphics with and without axes will be used next.

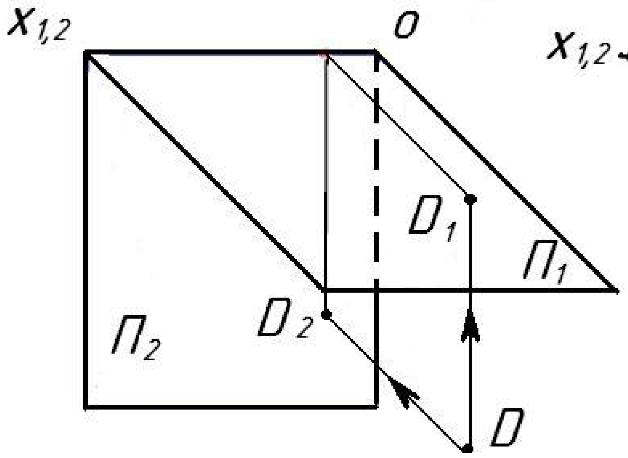


Fig.21

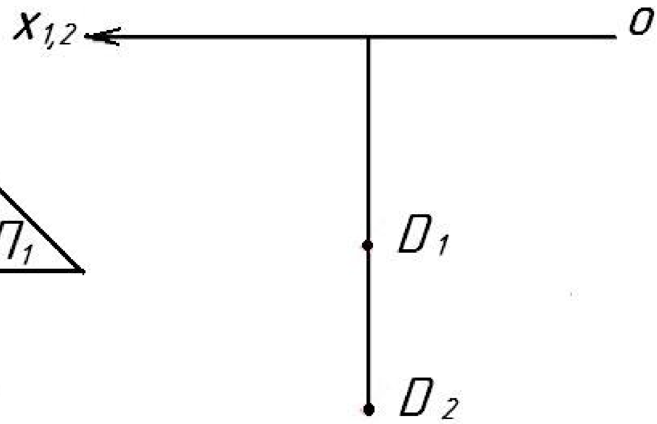


Fig.22

Questions to unit "A point"

1. How are planes Π_1, Π_2, Π_3 called?
2. What should one do to get a point projection?
3. How many point projections should one know to determine their position in space?
4. How many coordinates should be specified to find a point in space?
5. How is a point diagram made?
6. Construct points according to coordinates: A (2; 7; 4); B (0; 5; 6); C (8; 0;

Unit 3. A STRAIGHT LINE

As a position of a straight line in space is determined by its points, for the straight line construction it is necessary to construct projections of two points, which belong to the given straight line. Such points are extreme points of a straight line segment.

One straight line projection doesn't determine a straight line position in space. It is possible to draw several straight lines in plane α. Their projections coincide with the projection of straight line AB onto Π_1 (fig.23).

Two projections of a straight line completely determine its position in space.

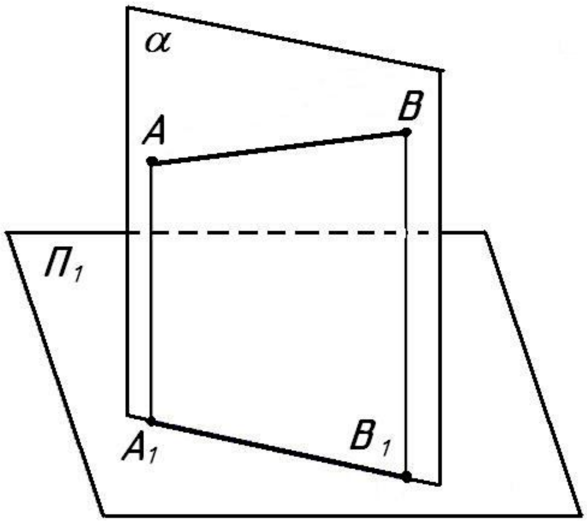


Fig.23

3.1. Projecting straight lines. Special position straight lines

The straight lines that are perpendicular to one of the projection planes are called projecting straight lines.

The name of the projecting straight line corresponds to the name of the projection plane it is perpendicular to.

A straight line that is perpendicular to horizontal projection plane Π_1 is called a horizontal projecting straight line (fig.24). A horizontal projection of such a straight line is a point. A horizontal projecting straight line is projected into its natural size onto a front and a profile projection planes.

A straight line that is perpendicular to front projection plane Π_2 is called a front projecting straight line (fig.25). A front projecting straight line is projected into a point onto a front projection plane. A horizontal and a profile projections of such a straight line correspond to its natural size.

A straight line that is perpendicular to profile projection plane Π_3 is called a profile projecting straight line (fig.26). A profile projecting straight line is projected into a point onto a profile projection plane. A horizontal and a front projections of this straight line correspond to a natural size of the straight line.

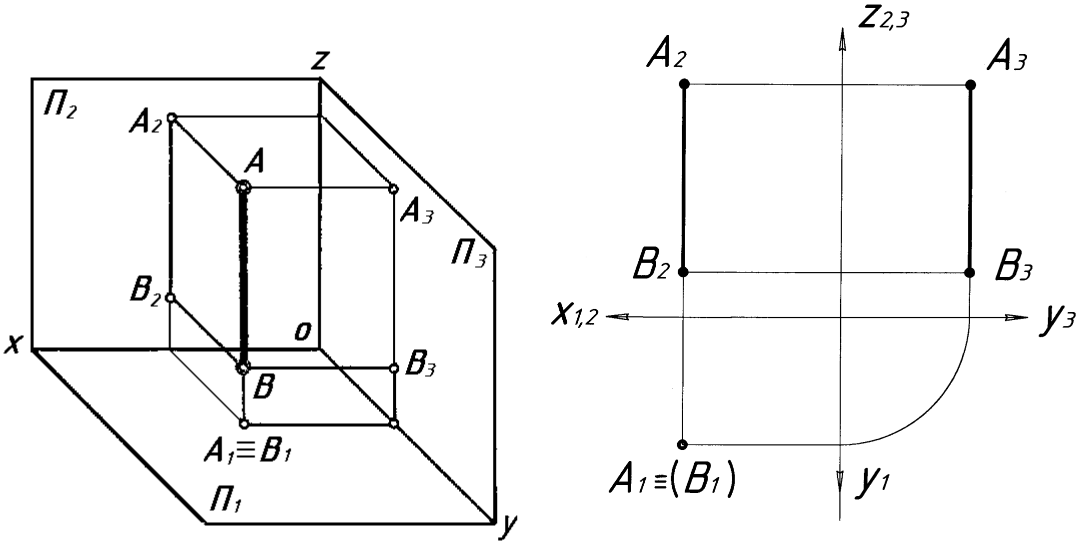
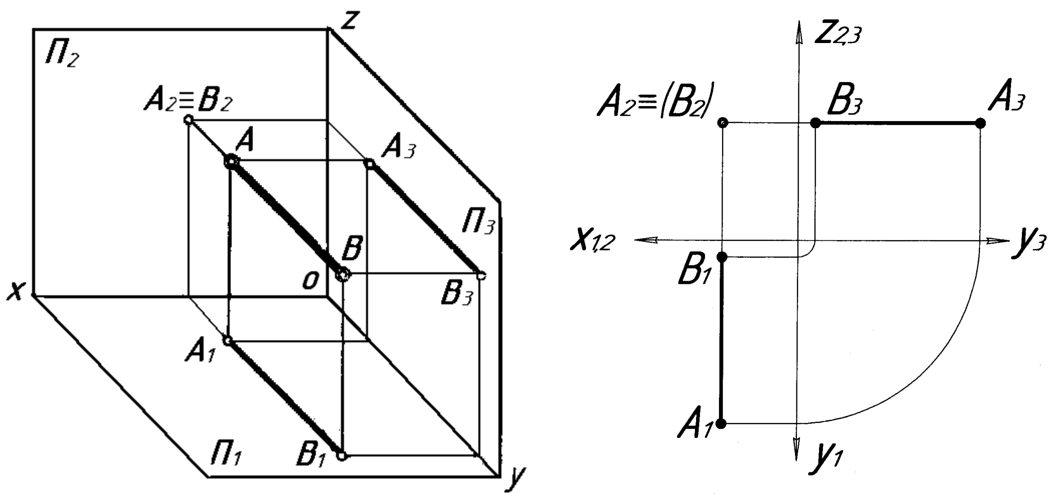

z
П2
A2
A
A3
П3
B2
o
x
B3
B
A1≡B1
П1
y
z2,3
A2
A3
B2
B3
x1,2
y3
A1≡(B1)
y1

Fig.24


z
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A2≡B2
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A1
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Fig.25

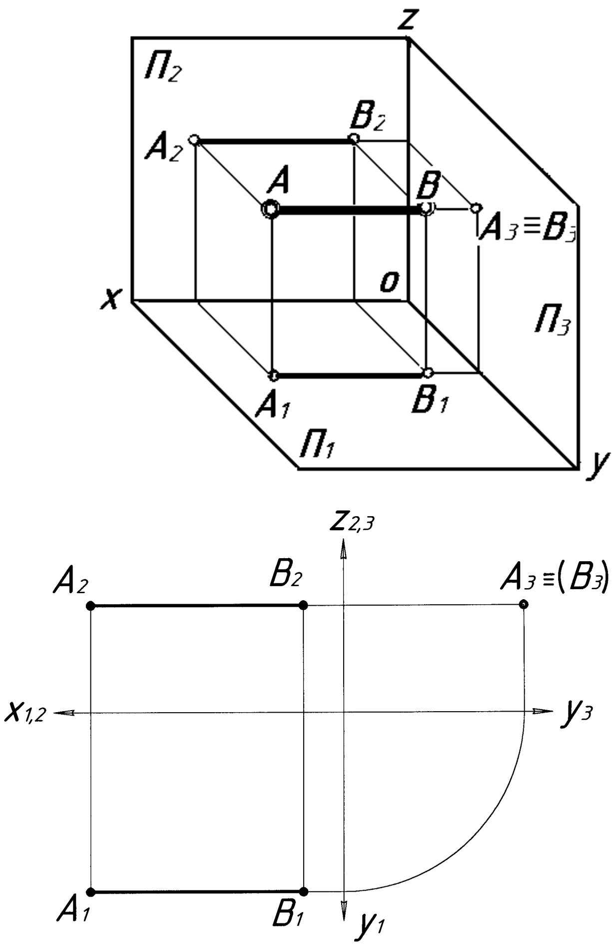
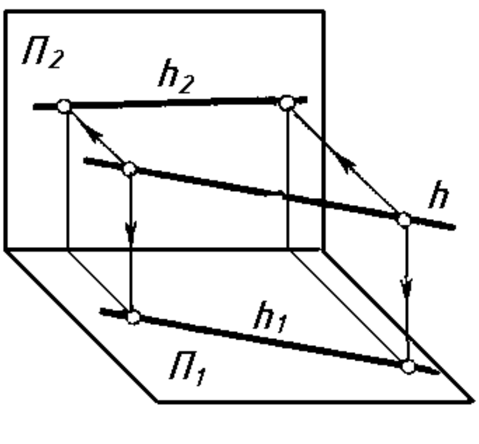


Fig.26

Level straight lines are the straight lines, parallel to one of the projection planes. The name of a straight line corresponds to the name of a projection plane.

A straight line, parallel to horizontal projection plane Π_1 is called a horizontal straight line and is marked with a letter *h* (fig.27).



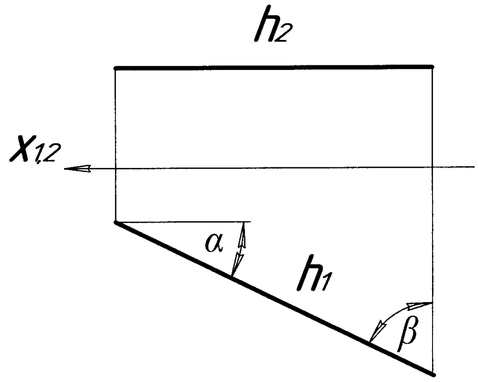


Fig.27

A straight line that is parallel to front projection plane Π_2 is called a front straight line and is marked with a letter *f* (fig.28).

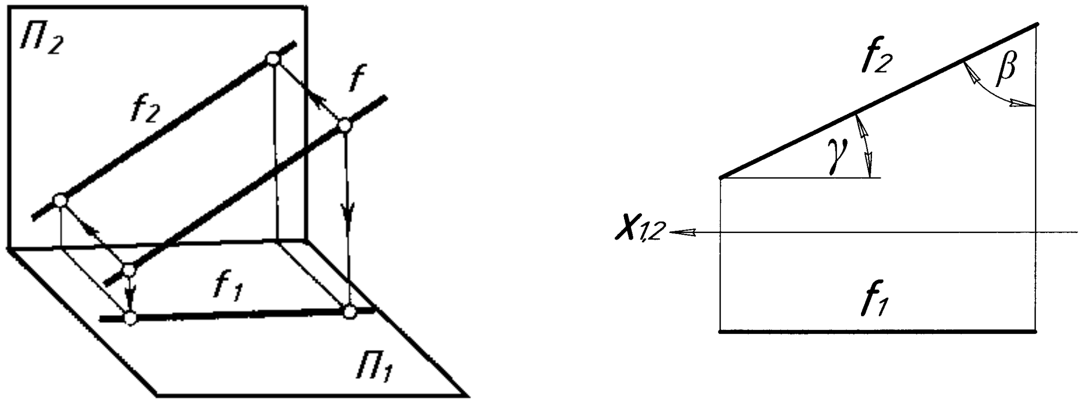


Fig.28

A straight line that is parallel to profile projection plane Π_3 is called a profile straight line and is marked with a letter *p* (fig.29).

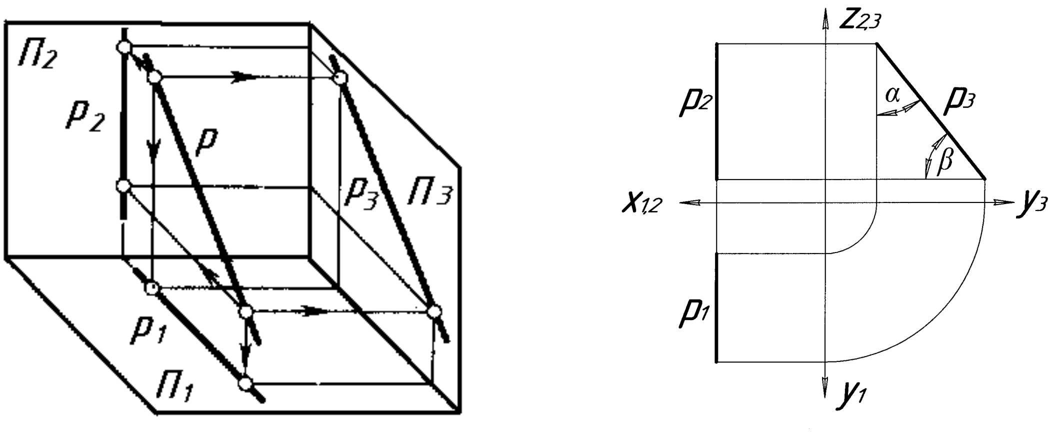


Fig.29

3.2. A general position straight line

A general position straight line is a line that is neither perpendicular, nor parallel to any of the projection planes (fig.30).

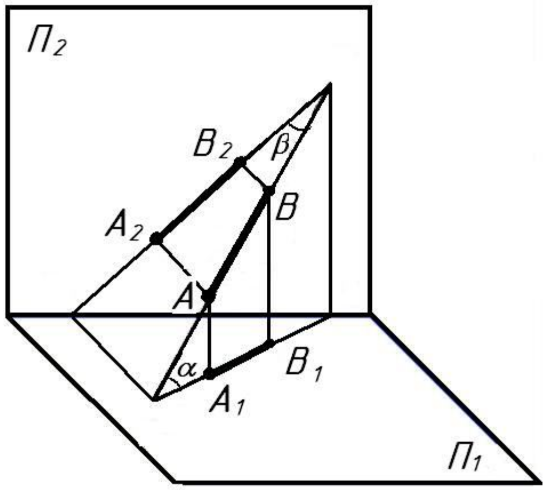


Fig.30

Straight line AB is a hypotenuse of right-angled triangle ABC; AC‖Π_2 and AC is perpendicular to Π_1. Cathetus AC is equal to the difference of coordinates Z of points A and B, i.e. AC=Za–Zb.

Angle α is an angle of inclination of AB to Π_1.

Therefore, in right-angled triangle ABD AD=A_2B_2, DB=Ya–Yb. Angle β is an angle of inclination of AB to Π_2.

3.3. A natural size of a general position segment

A natural size of a general position segment is equal to a hypotenuse of a right-angled triangle, one of the catheti of which is a horizontal (front) segment projection, another one is equal to the difference of coordinates Z (Y) of the segments ends (fig.31).

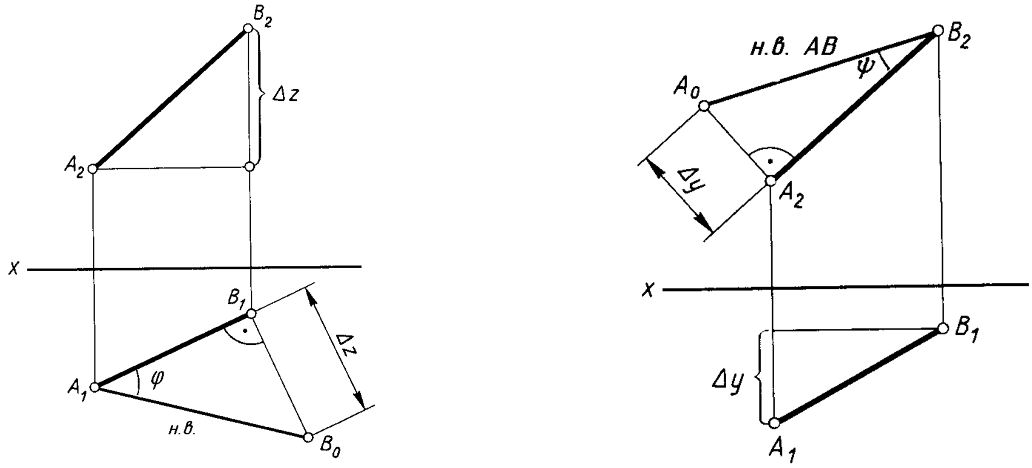


Fig.31

Angle α (β) between a horizontal (front) projection and a natural size of a segment is an angle of inclination of a segment to projection plane Π_1 (Π_2).

3.4. Straight line traces

A straight line trace is its intersection point with a projection plane (fig.30).

Point H is a horizontal trace of straight line AB, point F is a front trace of straight line AB.

To construct a front trace of a straight line on a diagram (fig.32), it is necessary to extend a horizontal projection of a straight line to its intersection with axis OX in point F_1 and from the obtained point to draw a perpendicular to its intersection with the extension of a front projection of a straight line in point $F_2 \equiv F$.

Point F_1 is a horizontal projection of a front trace and point F_2 is a front projection of a front trace. It coincides with the trace itself.

To construct a horizontal trace of a straight line it is necessary to extend a segment front projection to its intersection with the axis in point H_2 and from the obtained point to draw a perpendicular to its intersection with the extension of a horizontal projection of a straight line in point $H_1 \equiv H$. Point H_2 is a front projection of a front trace. It coincides with the trace itself.

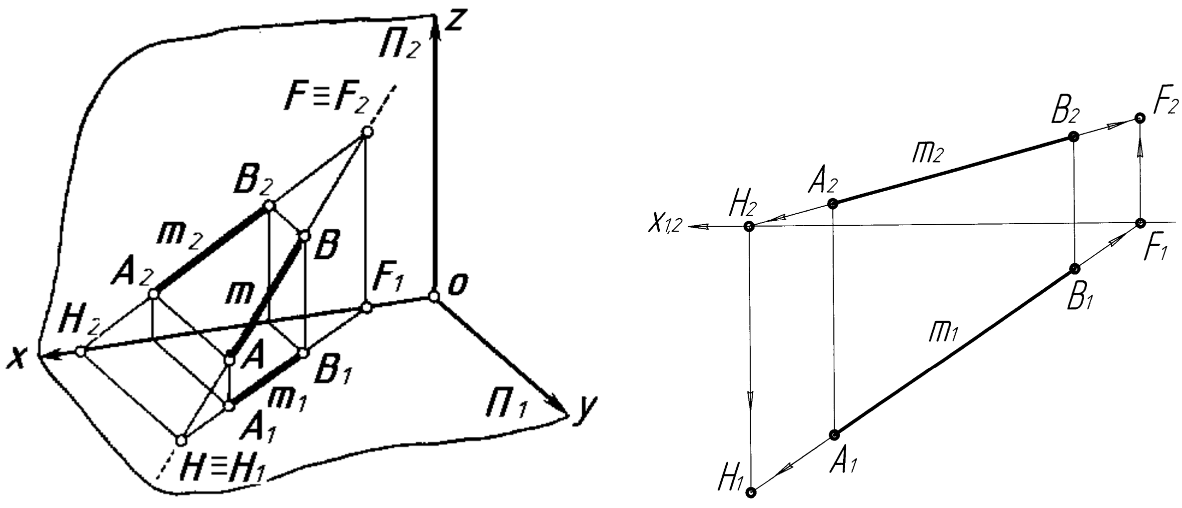


Fig.32

3.5. A point and a straight line

Let us examine geometric images in pairs to distinguish their positional and some metric properties. A point can or can not belong to a straight line.

If a point belongs to a straight line, this point projections lie in the uninamed projections of the straight line.

Fig.33 shows that points ACKB belong to a straight line, as its both projections belong to the corresponding straight line projections. Point D doesn’t lie on a given straight line, as its horizontal projection doesn’t coincide with the horizontal projection of the straight line. Point D is located in space above the straight line and in front of it.

The first condition of incidence:
$C_2 \in A_2B_2$; $C_1 \in A_1B_1 \rightarrow C \in AB$
$D_2 \in A_2B_2$; $D_1 \in A_1B_1 \rightarrow D \in AB$
$K_2 \in A_2B_2$; $K_1 \in A_1B_1 \rightarrow K \in AB$

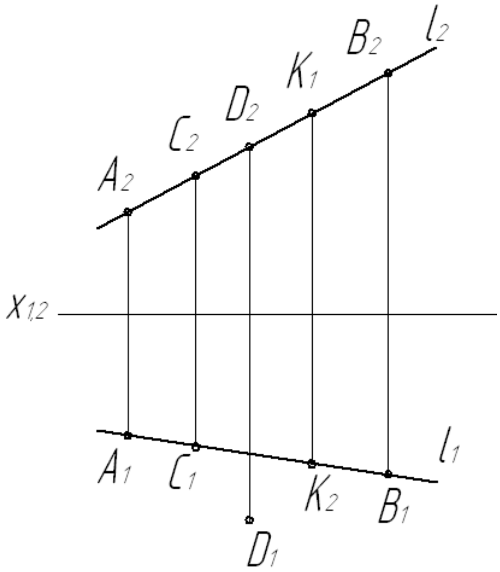


Fig.33

3.6. A mutual position of two straight lines
Two straight lines in space can have mutual position:
1) two parallel straight lines;
2) two intersecting straight lines;
3) two crosslying straight lines.

If two straight lines are parallel, their uninamed projections are parallel too (fig.34).

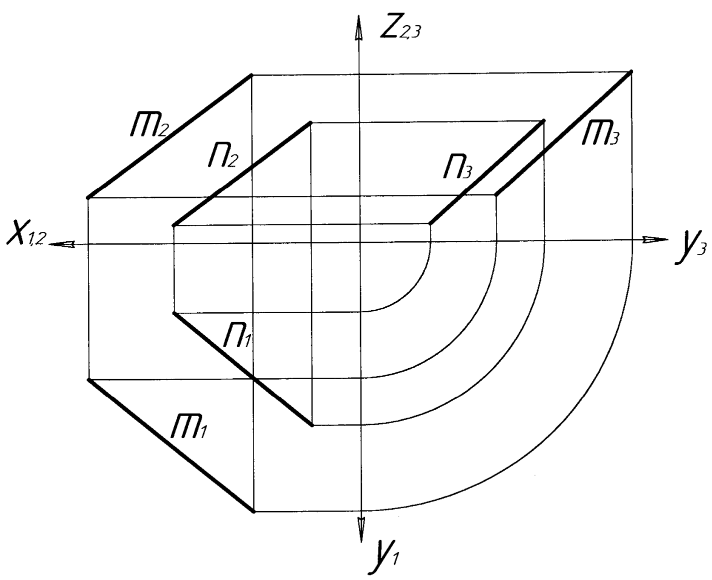


Fig.34

The parallel position of two profile straight lines is determined by their profile projections. If the straight lines intersect, their uninamed projections also intersect, and the intersection point projections lie on one link line (fig.35).

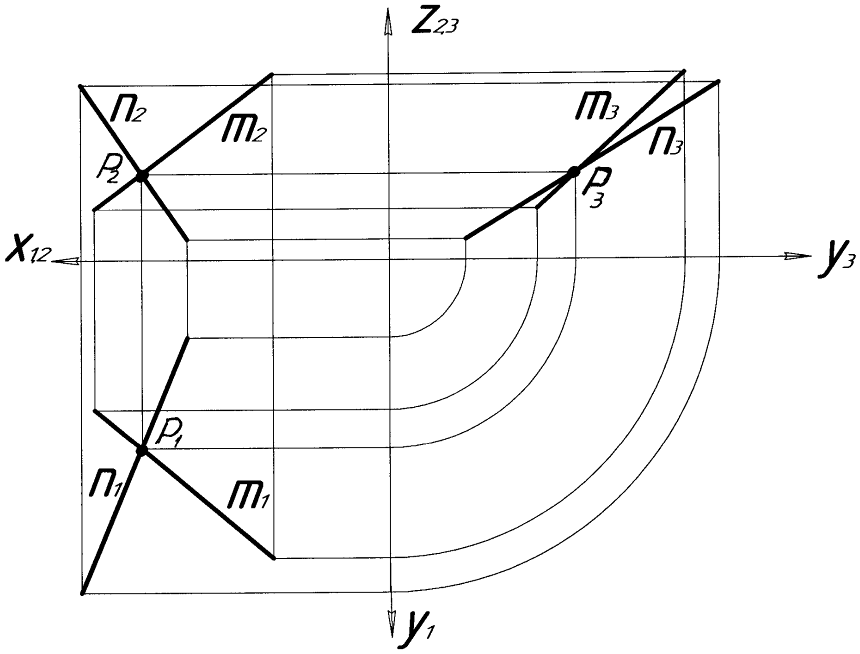


Fig.35

Two straight lines that are not parallel and don’t intersect each other are called the crosslying straight lines (fig.36). A pair of points A and B is called competitive, if their projections coincide on one of the projection planes.

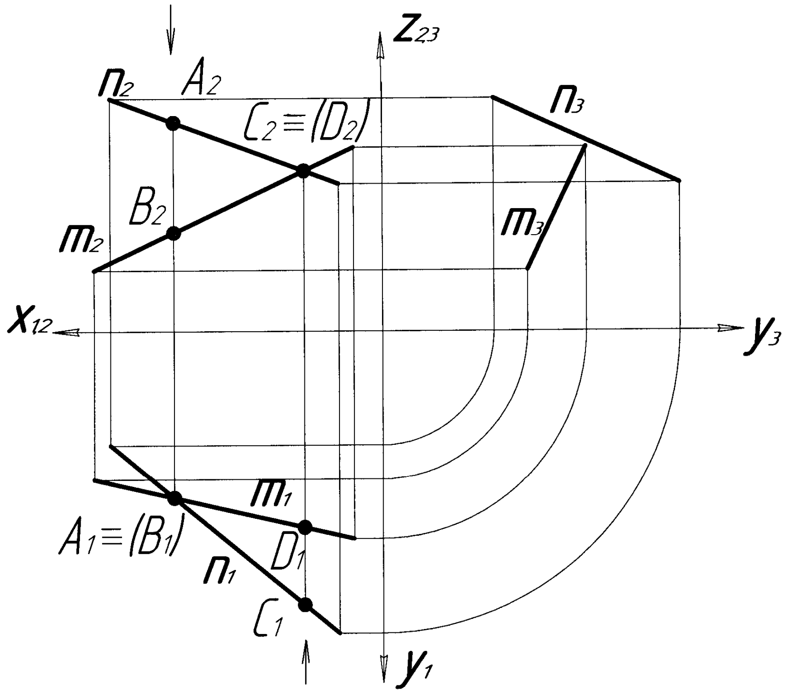


Fig.36

3.7. Right angle projection properties

If one side of a right angle is parallel to a projection plane, a right angle is projected onto this projection plane into its natural size (fig.37).

Fig.37 shows that AB Π_1, $\wedge A_1B_1C_1=90^0$

Problem. Determine the distance from point A to straight line *l* that is parallel to plane Π_1 (fig.38).

To determine the distance from point A to straight line *l* it is necessary to draw a perpendicular AC from point A to straight line *l*. As *l* is parallel to Π_1, a right angle between *l* and AC is projected onto Π_1 into its natural size. That's why we draw $A_1C_1 \perp l$, then we find A_2C_2 and with a right-angled triangle method we determine a natural size of AC.

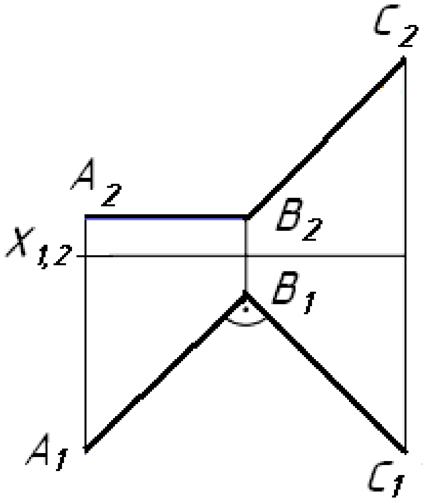


Fig.37

Fig.38

Questions to unit "A straight line"

1. How should one construct a projection of a straight line segment?
2. Which straight lines are called the level straight lines?
3. Construct the projections of a horizontal, a front, a profile straight line.
4. Which straight lines are called projecting straight lines?
5. Construct the projections of the horizontal, front, profile projecting straight lines.
6. Specify the algorithm to find a natural size of a straight line with a right-angled triangle method.
7. How can one find the angles of inclination of a general position straight line to projection planes Π_1 and Π_2?
8. What is called a straight line trace?
9. How are the projections of two parallel straight lines, two intersecting straight lines, two crosslying straight lines placed?
10. What is a right angle projection property?
11. In which case the distance between the straight lines of a general position is projected into its natural size?

Unit 4. A PLANE. METHODS OF ITS DEPICTING

4.1. Methods of plane depicting

A plane can be specified by six methods. Let's study them.

1. A plane can be drawn through three points, which don't lie on one straight line (fig.39).
2. A plane can be drawn through a straight line and a point that doesn't lie on this straight line (fig.40).
3. A plane can be drawn through two parallel straight lines (fig.41).
4. A plane can be drawn through two intersecting straight lines (fig.42).

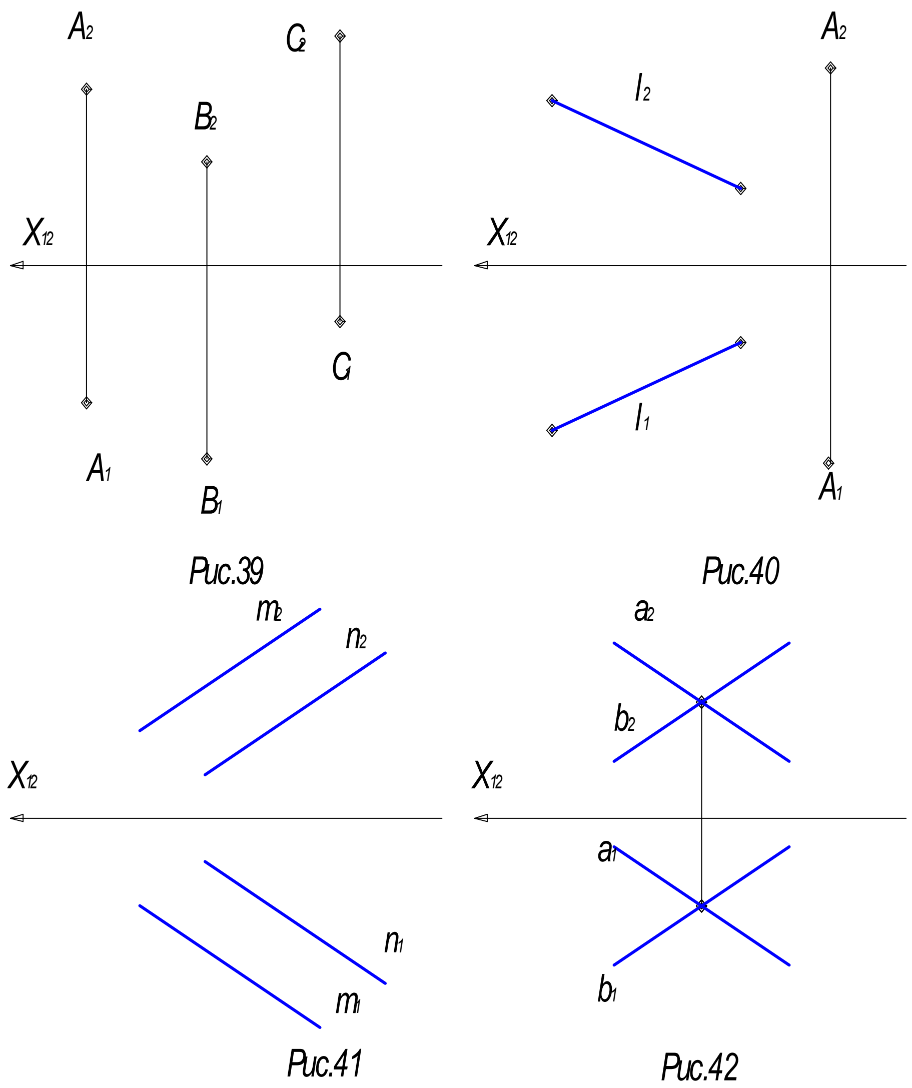


Fig. 39, 40, 41, 42

5. A plane is specified by a cut off of any form (fig.43).

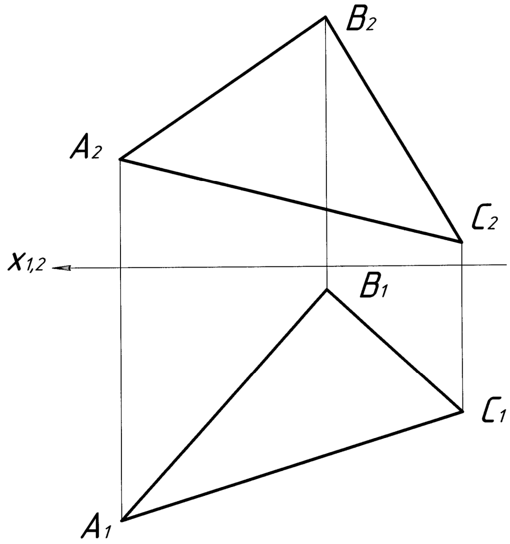


Fig.43

6. A plane is specified by traces (fig.44).

A straight line on which a plane intersects a projection plane is called a plane trace.

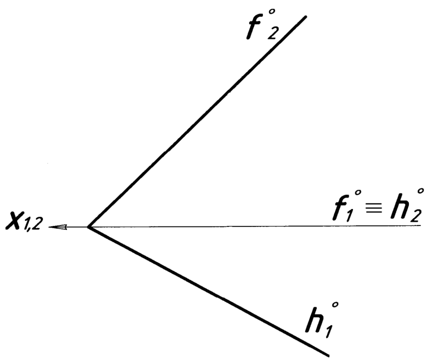


Fig.44

A plane intersection with Π_1 is called a horizontal trace and with Π_2 – a front trace.

Fig.44 shows a general position plane that is specified by traces. A front trace of plane L coincides with its front projection l_2, and a horizontal trace of plane K coincides with its horizontal projection k_1. A horizontal projection of front trace l_1 coincides with a front projection of horizontal trace k_2 and lies on axis OX_{12}.

If a projection plane has been specified, one of its traces is depicted athwart axis X_{12}. This trace in most problems is not shown on projections.

4.2. A plane location in space. Special position planes

Special position planes are projecting planes and level planes. Projecting planes are the planes that are perpendicular to one of the projection planes. Their name corresponds to the name of the projection plane, which they are perpendicular to.

A plane that is perpendicular to Π_1 is called a horizontal projecting plane; a plane that is perpendicular to Π_2 is a front projecting plane; a plane that is perpendicular to Π_3 is a profile projecting plane.

Horizontal projections of all the points that belong to a horizontal projecting plane lie on one straight line, which is a horizontal trace of the given plane (fig.45).

This property of a horizontal trace of a horizontal projecting plane is called a collecting property.

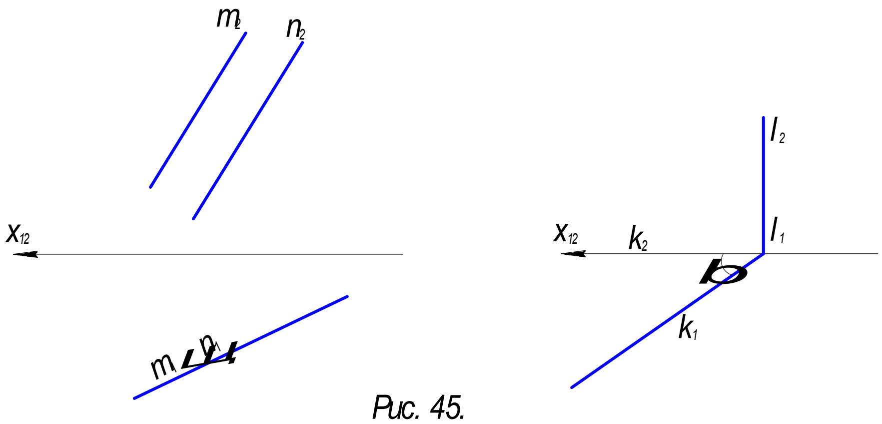


Рис. 45.

Fig.45

A front trace of a front projecting plane and a profile trace of a profile projecting plane have the similar property. Level planes are the planes that are parallel to one of the projection planes. Planes that are parallel to Π_1 are called horizontal planes (fig.46), a front trace of such a plane is parallel to axis X_{12}.

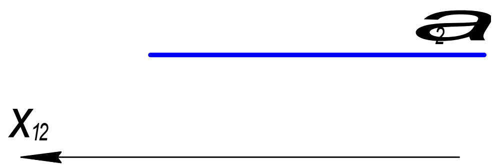


Рис. 46.

Fig.46

Planes that are parallel to Π_2 are called front planes. A horizontal trace of a front plane is parallel to axis X_{12} (fig.47).

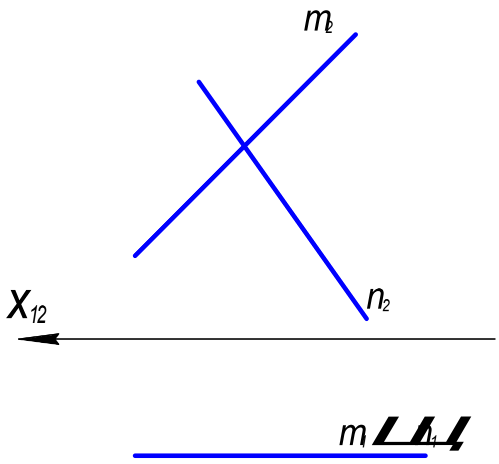


Рис. 47.

Fig.47

Planes that are parallel to Π_3 are called profile planes (fig.48).

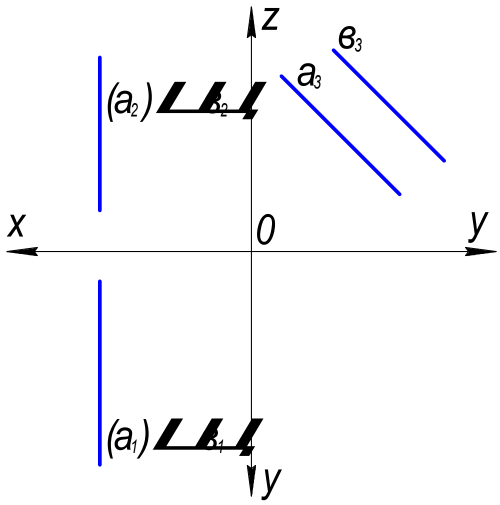


Рис. 48.

Fig.48

We offer you yourself to construct projections of the front projecting and the profile projecting planes that are specified by any method.

A trace of level planes as the traces of the projecting planes are characterized by the collecting property.

4.3. A general position plane

A general position plane is a plane that is neither parallel, nor perpendicular to any of the projection planes (fig.39-44).

4.4. A point and a straight line on a plane

A straight line belongs to a plane, if it has two common points with it (fig.49).

A point belongs to a plane, if it lies on a straight line, which belongs to the given plane (fig.49).

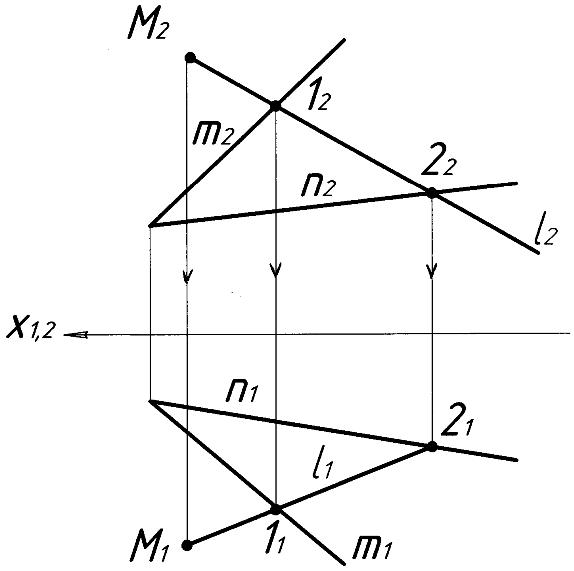


Fig.49

A straight line belongs to a plane, which is specified by $m \cap n$, because it has two common points 1and 2 with it.

Point M belongs to plane $m \cap n$, because it lies on straight line *l*, which belongs to the given plane.

4.5. A parallel position of a straight line and a plane

A straight line is parallel to a plane, if this straight line is parallel at least to one straight line that belongs to the given plane.

Problem. Draw a straight line through point A that is parallel to the specified plane (fig.50).

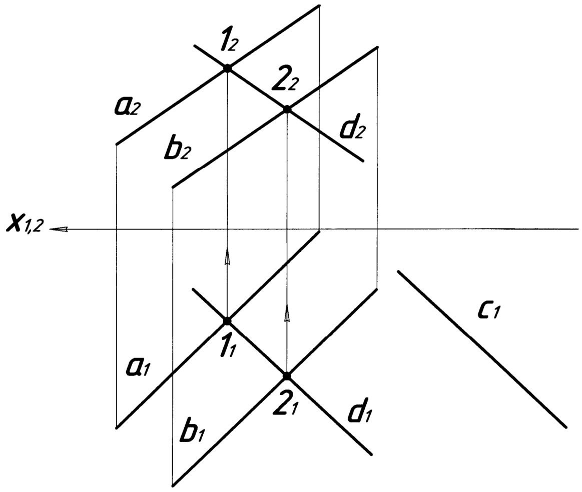


Рисунок 4.19

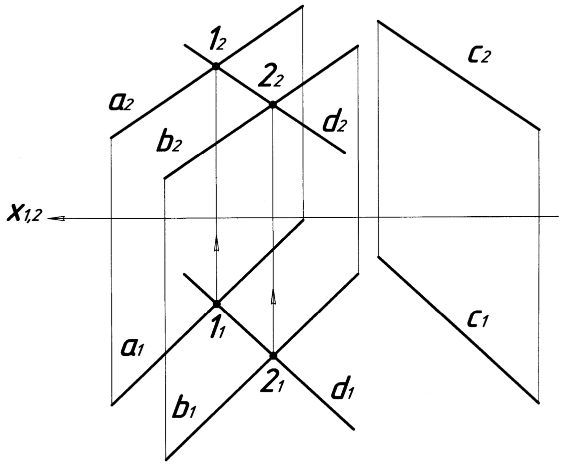


Рисунок 4.20

Fig.50

Solving. We draw any straight line *l* that belongs to plane (a‖b) in a plane that is specified by two parallel straight lines (a‖b). Next we construct straight line *c*. It goes through point A and it is parallel to straight line *d*, and thus, to plane (m‖n).

4.6. Special lines of a plane

A horizontal line, a front line and a line of the largest inclination belong to special lines of a plane.

Straight line h_1 that belongs to the given plane and is parallel to horizontal projection plane Π_1 is a horizontal line of a plane.

Straight line f_1 that also belongs to the given plane and is parallel to front projection plane Π_2 is a front line of a plane (fig.51).

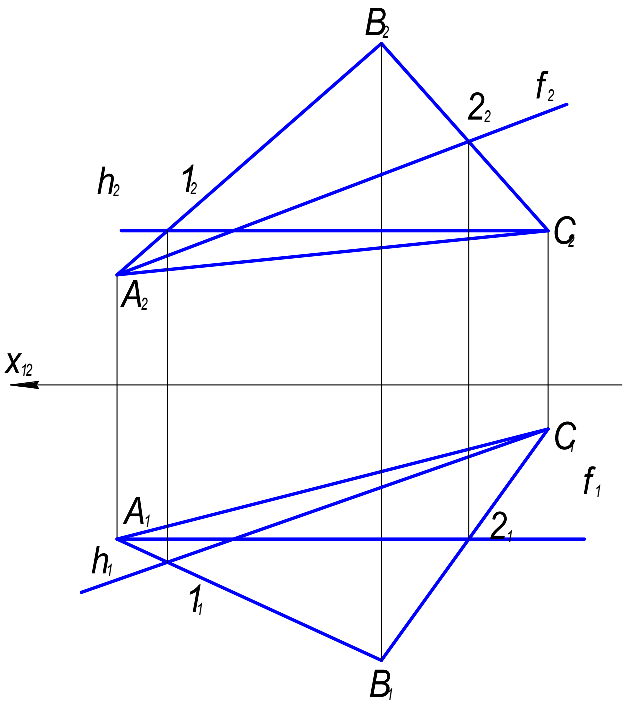


Рис. 51.

Fig.51

Straight line h_2 is a front projection of a horizontal line; straight line h_1 is a horizontal projection of a horizontal line. Straight line f_2 is a front projection of a front line; straight line f_1 is a horizontal projection of a front line.

A straight line that belongs to the given plane and is perpendicular to its trace is called a line of the largest inclination of a plane.

A line of the largest inclination in relation to Π_1 is called a line of the largest slope. It is perpendicular to a horizontal trace of the given plane or to its horizontal line. An angle of the inclination line of the largest slope to Π_1 is an angle of inclination of the given plane to Π_1.

A line of the largest inclination in relation to Π_2 is perpendicular to a front trace of the plane or to its front line. An angle between the line of the largest inclination and Π_2 is an angle of inclination of the given plane to Π_2.

Problem. Determine an angle of inclination of the given plane to Π_1 (fig.52).

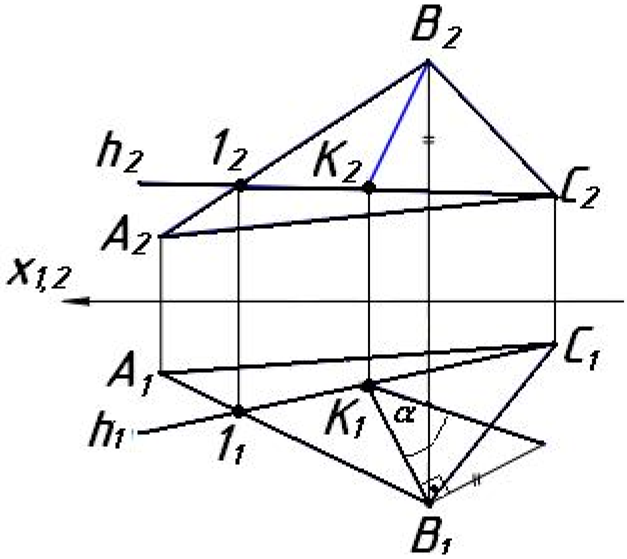


Fig.52

Solving. To solve this problem, it is necessary to construct in the specified plane the projections of lines of the largest inclination to Π_1 and to determine an angle of its inclination to Π_1.

1. We draw horizontal line *h* in plane ABC.

2. Then we put down a perpendicular to h_1 on a horizontal projection. It is more convenient to draw it from point B_1. Line BK is a line of the largest inclination to Π_1.

3. To determine angle α we use a right-angled triangle method.

To determine angle β of inclination of the given plane to Π_2, it is necessary to construct a line of the largest inclination to Π_2.

Questions to unit "A plane. Methods of its depicting"

1. Which methods can one use to depict a plane in space?
2. How can a plane be located in space?
3. Which planes are called projecting planes?
4. Which plane is called a general position plane?
5. Formulate a definition of a point that belongs to a plane.
6. Formulate a definition of a straight line that belongs to a plane.
7. Which lines are called special lines of a plane?
8. How can one construct a line of the largest inclination of a plane?

Unit 5. MUTUAL POSITION OF TWO PLANES

Planes can have the following positions in relation to each other:

1) planes are parallel to each other;

2) planes intersect;

3) planes are mutually perpendicular.

5.1. A parallel position of two planes

Two planes are parallel, if two intersecting straight lines of one plane are parallel to two intersecting straight lines of another plane (fig.53).

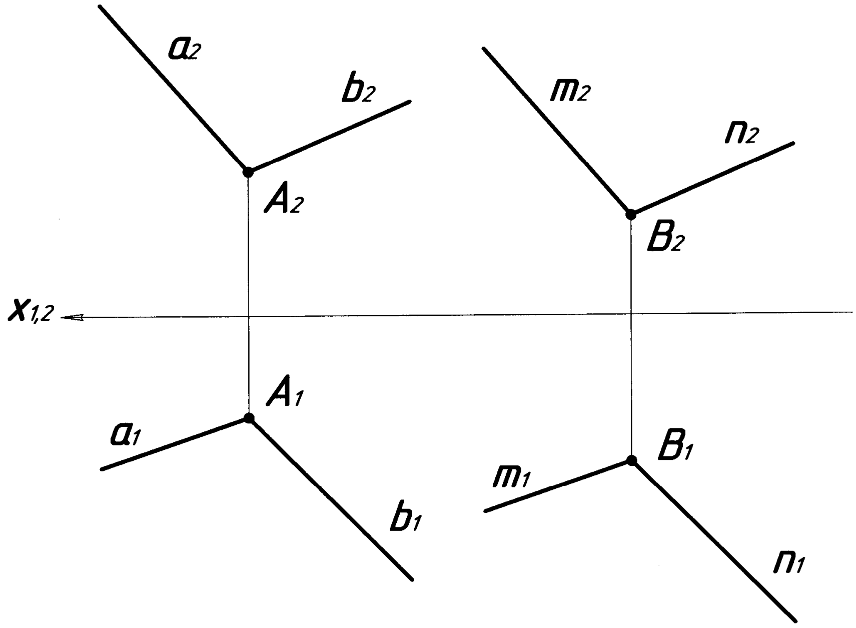


Fig.53

5.2. Intersection of two planes

Problem 1. Two planes intersect in a straight line, the position of which is specified by two points.

It is necessary to find two points that are common for both planes and to connect them.

A. Two planes are projecting ones (fig.54).

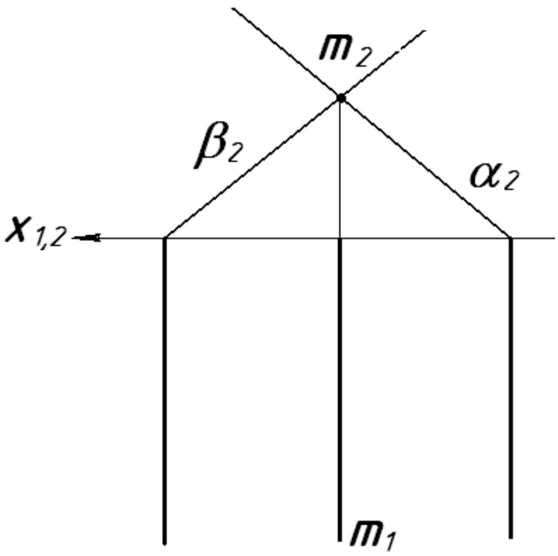


Fig.54

Two front projecting planes intersect in this problem. Their intersection line is a front projecting straight line.

So, if two projecting planes of the same name intersect, then an intersection line is a projecting straight line. In this case to construct an intersection line, it is enough to determine the position of one point and to know the direction of an intersection line.

B. One plane is a projecting one, another plane is a general position plane (fig.55).

One plane is specified by triangle ABC, another one – by horizontal projecting plane α in this problem.

A horizontal projection of intersection line (1–2) coincides with horizontal trace α_1 and belongs to triangle ABC. Then we find a front projection of intersection line (1–2).

Therefore, if one of the planes which is intersecting is also a projecting one, a projection of an intersection line of the planes coincides with a projection of a projecting plane. The only one thing that is left to do is to construct another projection of a straight line of intersection.

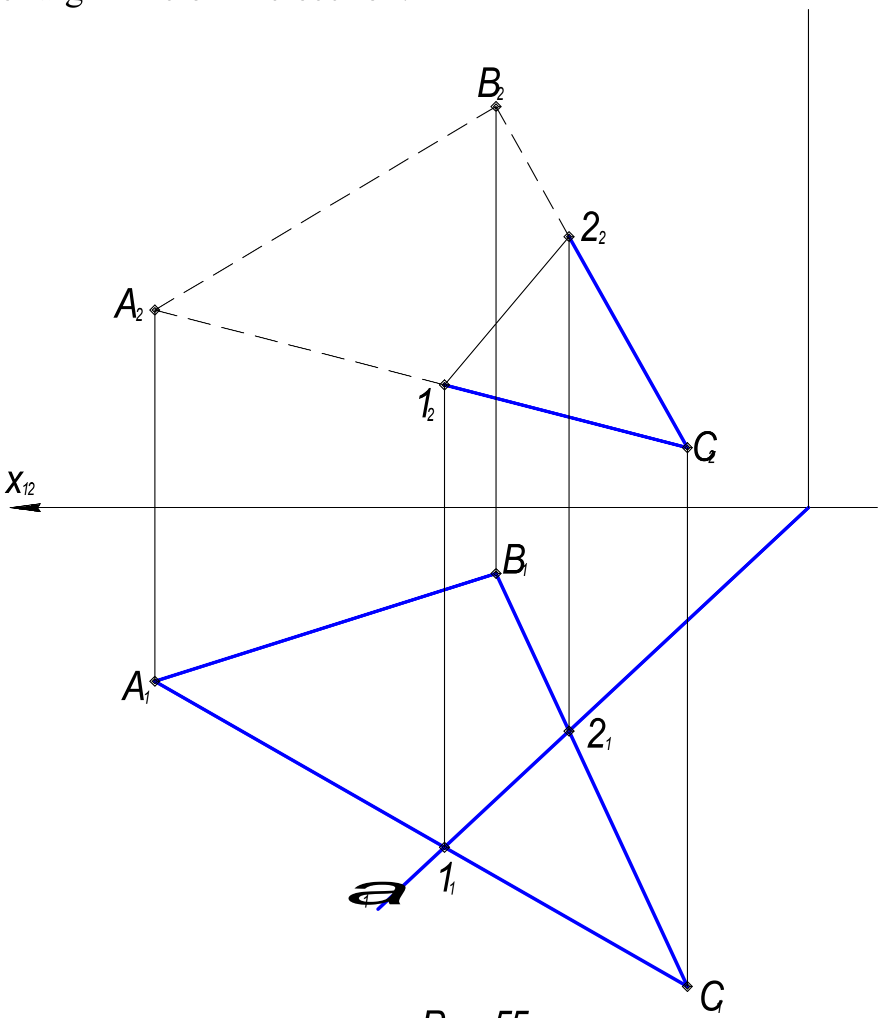


Рис. 55.

Fig.55

C. Two general position planes (fig.56).

The intersection line is determined by a method of auxiliary sections which are drawn with the help of the level planes or projecting planes.

Solving. 1. We intersect both general position planes by auxiliary horizontal plane α.

2. We construct intersection line (1–2) of plane α and plane (m‖n).

3. Then we construct intersection line (3–4) of plane α and plane ABC.

4. On the extension of horizontal projections of intersection lines we mark point K_1 that is a horizontal projection of the point which belongs to three planes at the same time. We find a front projection of point K_2.

5. We intersect both general position planes by auxiliary plane β and find another intersection line point – point L.

6. Having connected points K and L, we shall get projections of intersection line KL.

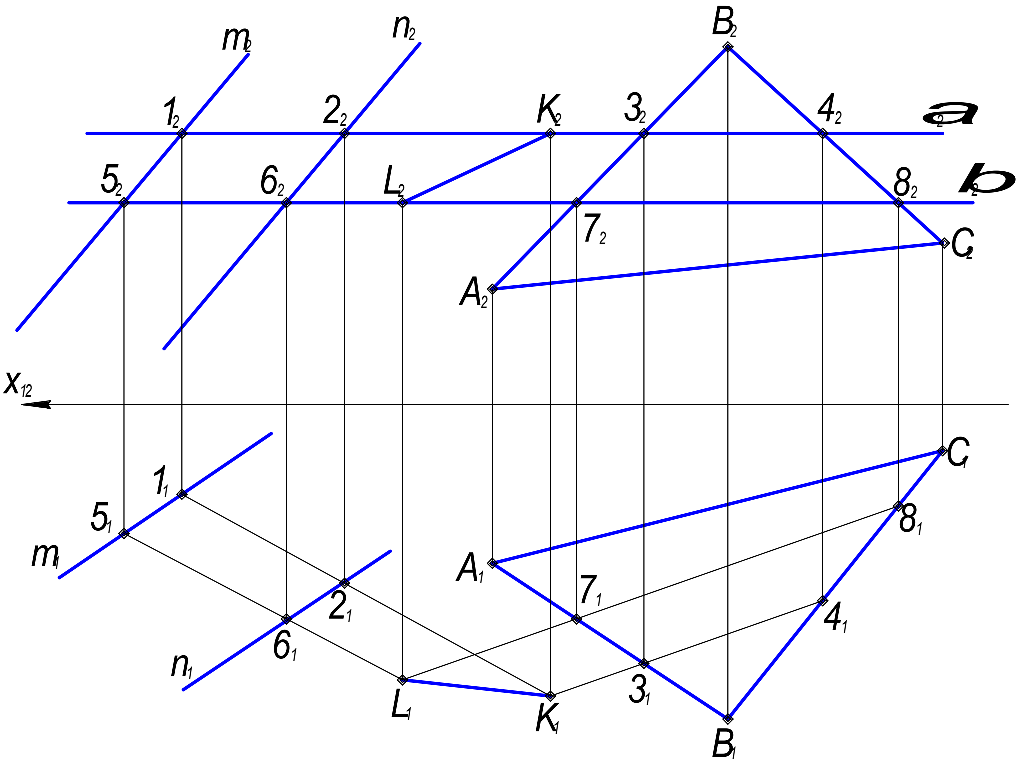


Рис. 56.

Fig.56

Problem 2. Construct projections of the intersection lines of two general position planes that are specified by traces (fig.57).

Solving. If two planes are specified by traces, two common points which belong to an intersection line will become intersection points of traces (points 1, 2). Having connected the uninamed projections of these points, we shall get the projections of the intersection lines of the given planes.

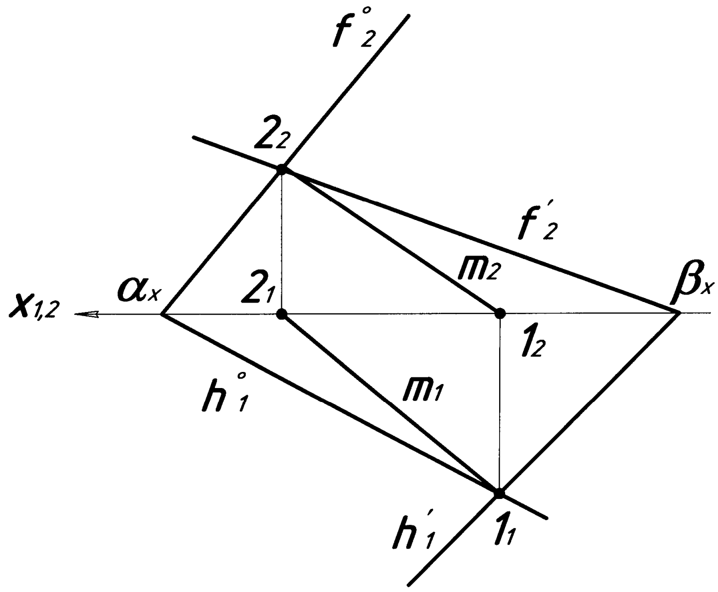


Fig.57

5.3. Intersection of a straight line and a plane

To construct an intersection point of straight line *l* and a plane, it is necessary to do the following steps:

1. To draw a plane, a projecting one is advisable, through straight line *l*.
2. To construct an intersection line of the given plane and a projecting plane.
3. To mark the searched intersection point of straight line *l* and a plane where straight line *l* intersects the constructed line of two planes intersection.

In problems to search an intersection point of a straight line and a plane it is necessary to determine visibility of a straight line in relation to the specified plane by a method of competitive points.

Problem 1. Construct projections of an intersection point of straight line *a* and plane ABC (fig.58).

Solving. 1. Through straight line *a* we draw an auxiliary plane in a special position. Plane α is a horizontal projecting plane in this problem.

2. We find projections of an intersection line of auxiliary plane α and a specified plane (line 1–2).

3. We determine an intersection point of a straight line and a plane – point K.

4. With the help of competitive points we determine visibility.

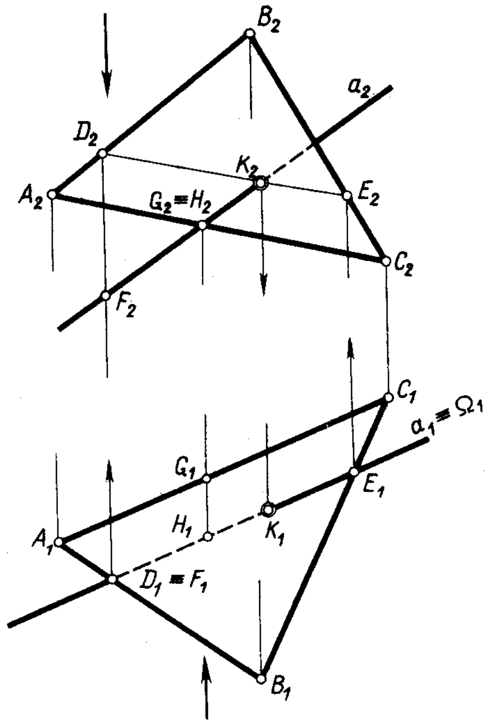


Fig.58

Problem 2. Construct an intersection point of straight line *a* and a plane (fig.59).

Solving. 1. We include a specified straight line in a front projecting plane.

2. We build an intersection line of a specified plane and a front projecting plane.

3. Then we mark intersection point K, where the constructed intersection line intersects the straight line.

4. By a method of competitive points we determine visibility of the straight line in relation to the specified plane.

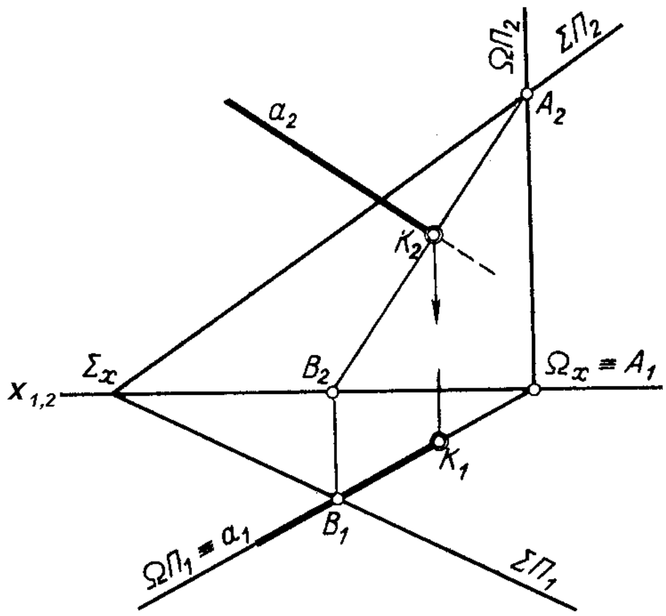


Fig.59

5.4. Perpendicularity of a straight line and a plane

A straight line is perpendicular to a plane, if it is perpendicular to two straight lines in a plane that intersect each other.

Two straight lines that intersect will be a horizontal and a front line of a plane.

Problem. From point A draw a perpendicular to a plane that is specified by triangle BCD (fig.60).

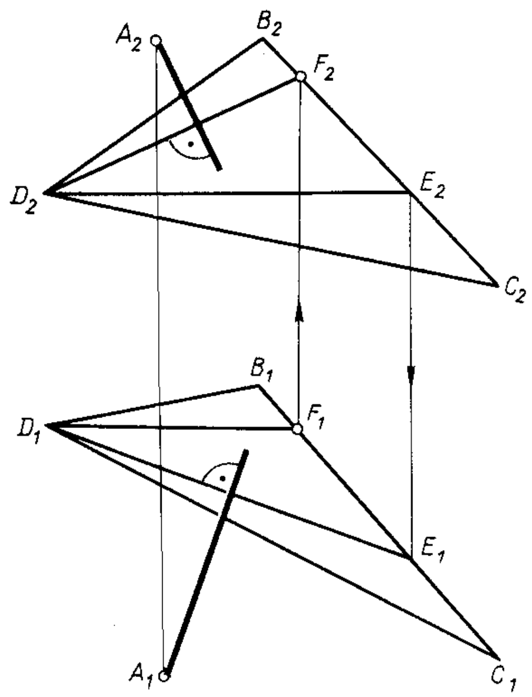


Fig.60

To solve the problem, we draw projections of horizontal line *DE* and front line *DF* in a plane. According to the property of a right angle projection we draw a front projection of a perpendicular from point A at a right angle to D_2F_2 and a horizontal projection – from point A_1 at a right angle to D_1E_1.

5.5. Mutually perpendicular planes

Planes are mutually perpendicular, if one of them goes through a perpendicular to another plane.

Problem. Through point A draw projections of a plane that is perpendicular to the specified plane ($h \times f$) (fig.61).

Solving. From point A_1 we draw straight line n_1 athwart h_1. From point A_2 we draw straight line n_2 athwart f_2. We construct projections of straight line *m* at random. This way we specify a perpendicular plane by two straight lines that intersect ($n \times m$).

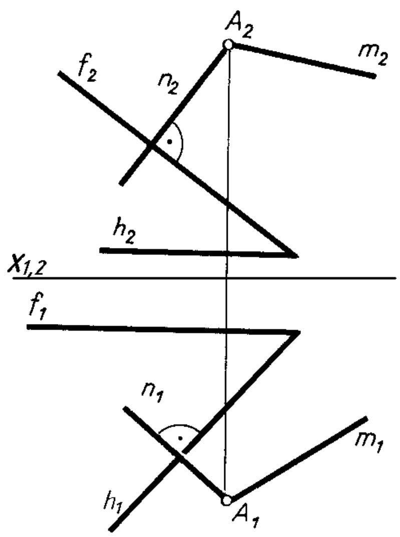


Fig.61

Questions to unit "Mutual position of two planes"

1. Formulate a definition of a parallel position of two planes.
2. How does one construct an intersection line of two planes, if one of them is a projecting one?
3. How should one construct an intersection line of two planes, if both of them are general position planes?
4. How can one draw a projecting plane through a straight line?
5. Formulate a rule to find an intersection point of a straight line and a plane?
6. How does one determine visibility of a straight line in relation to a projecting plane?
7. What is the indication of perpendicularity of a straight line to a plane?

8. What is the indication of perpendicularity of two planes?

Unit 6. A PROJECTION PLANE REPLACEMENT METHOD

The essence of a projection plane replacement method is that a position of the depicted points, lines, plane figures in space remains constant, and the system of planes Π_1, Π_2 is supplemented by new planes that make up from Π_2 and Π_1 or with each other the systems of two mutually perpendicular planes which are considered to be projection planes.

Every new system of projection planes is selected to get a position that is the most convenient to make the necessary construction.

The use of a projection plane replacement method for solving different problems is based on four main problems.

Problem 1. Make straight line *l* of a general position a level line in a new system of projection planes.

We will specify on a graphic straight line *l* of a general position by segment AB (fig.62). Using the possibility to choose the position of a projection axis – "a base of counting the distances", one can draw on a complex graphic this axis (X_{12}) through point A_2 that has the smallest height.

Fig.62 shows that straight line *a* is not a level line, because none of its projections is parallel to axis X_{12}. That is why to make straight line *a* a level line, for example a front line, in relation to a new projection plane parallel to *a*, we draw horizontal projecting plane Π_4 and we move from system ($\Pi_1 \perp \Pi_2$) to system ($\Pi_1 \perp \Pi_4$). A new projection axis should be parallel to a_1.

To construct a new front projection of straight line *a* we draw new link lines athwart X_{14} and mark new projection points A and B on them, point A_4 – on axis X_{14} and point B_4 on the height *h*. Having connected the found points, we shall get a new projection of straight line a_4 (A_4B_4).

Therefore, straight line *a* in a new system of projection planes ($\Pi_1 \perp \Pi_4$) is a front level line, as $a_1 \parallel X_{14}$, so $a_1 \parallel \Pi_4$. That is why segment A_4B_4 is equal to natural segment AB.

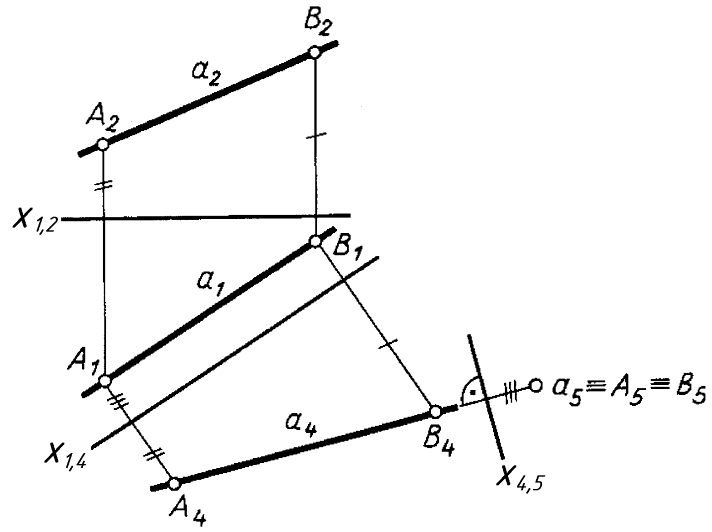


Fig.62

Thus, after replacement of plane $П_2$ by plane $П_4$ the following things have been reached:

1. Straight line *a* (a_1, a_2) became a level line;
2. Segment A_4B_4 is equal to natural segment AB;
3. Angle α that has been made up by a projection of A_4B_4 and axis X_{14} is equal to a natural size of an angle of straight line *a* (AB) to horizontal projection plane $П_1$.

Problem 2. Make straight line *l* of a general position in a new projection plane system a projecting one.

We have already examined the transformation of a general position straight line into a level line with the help of a projection plane replacement method (fig.63).

To transform straight line *l* (AB) to a projecting line, it is necessary to replace one more projection plane, if we move from system ($П_1 \perp П_4$) to system ($П_4 \perp П_5$).

We draw new projection plane $П_5$ athwart projection plane $П_4$ and besides, athwart straight line $AB \parallel П_4$, so that straight line AB will become a projecting line ($П_5 \perp AB$).

It is necessary to draw a new projection axis on a graphic (fig.62) athwart A_4B_4 ($X \perp A_4B_4$). So, link lines A_4A_5 and B_4B_5 will coincide with straight line A_4B_4 in this case. Putting segment *m* on the link line from new axis X_{45}, we'll get a projection of a specified straight line onto plane $П_5$ as point $l_5 \equiv A_5 \equiv B_5$.

Therefore, after we do another projection plane replacement, we'll move to system ($П_4 \perp П_5$). Straight line AB becomes a projecting line in relation to plane $П_5$ as one point $A_5 \equiv B_5$.

Problem 3. Make plane α (ABC) of a general position a projecting one with the help of a projecting plane replacement method.

To make plane α (DEF) a projecting one, we replace plane Π_2 by new projection plane Π_4, drawing the latter one athwart α (DEF) (fig.63).

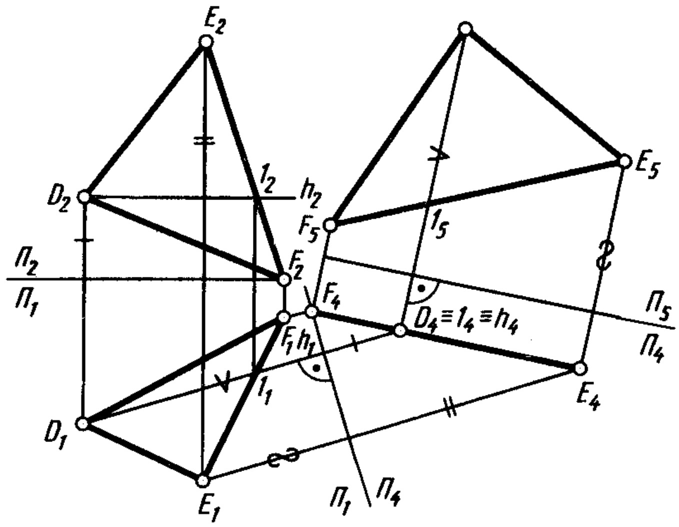


Fig.63

To do this we draw horizontal line h in plane α (DEF). We specify new projection plane Π_4 athwart this horizontal line, so athwart projection plane Π_1. Then horizontal line h and plane α (DEF) will become projecting in relation to Π_4.

To make the replacement on a complex graphic we draw new axis X_{14} athwart a horizontal projection of horizontal line h_1 ($X_{14} \perp h_1$). From each point (D_1, E_1, F_1) we draw link lines onto Π_4 athwart X_{14}. On these lines we put coordinates of points, which we take from Π_2. Then we have new projections of points D_4, E_4, F_4, that are located on one straight line – new projection of plane α (DEF).

So, replacing plane Π_2 by plane Π_4, we reach the following things:

1. Plane DEF has become a projecting one;

2. Angle α, made by projection $D_4E_4F_4$ and axis X_{14} is equal to a natural size of an inclination angle of the specified plane to horizontal projection plane Π_1.

Problem 4. Make plane α (ABC) of a general position a level plane by a projection plane replacement method.

We have already examined the transformation of a general position plane into a projecting plane by a projection plane replacement method (fig.63).

To transform plane α (ABC) into a level plane from projection system ($\Pi_1 \perp \Pi_4$), it is necessary to move to new system ($\Pi_4 \perp \Pi_5$), i.e. additionally to replace plane Π_1 by new plane $\Pi_5 \parallel \alpha$ (ABC). In order to do it, we draw new axis X_{45} which is parallel to α_4 (A_4, B_4, C_4) on any distance from the latter one. From each point A, B, C we draw link lines onto Π_5 athwart X_{45}. From axis X_{45} we put coordinates of the corresponding points from Π_1 to axis X_{14} on link lines. Thus, we will have projections of points A_5, B_5, C_5 on Π_5. Having connected them, we make up a new projection of triangle ABC on Π_5 (fig.63).

So, replacing both projection planes gradually, we shall reach the following things:

1. Plane ABC has become a level plane in relation to projection plane Π_2;
2. Projection ($A_5B_5C_5$) is equal to a natural size of triangle ABC.

6.1. Examples of solving some problems due to a projection plane replacement method

Problem 1. Determine the distance from point A to plane α. On fig.64 plane α of a general position is specified by traces.

Solving. We draw an additional projection plane athwart trace *k*, i.e. to an intersection line of the given plane and Π_1. So, plane Π_4 will be perpendicular to Π_1 and to the specified plane α. To get a trace of plane α on Π_4 we choose arbitrary point l_2 on trace l_2 and we transfer it onto Π_4. Having connected a point of traces superposition and point l_4, we shall get a trace of plane α which is perpendicular to Π_4.

Then we put a segment on a link line from point M_1 on Π_4 from axis X_{14}. The segment should be equal to a segment from point M_2 to axis X_{12} and we shall get point M_4. The searched distance from point M to plane α is determined by a perpendicular which is drawn from point M_4 onto a trace of plane α.

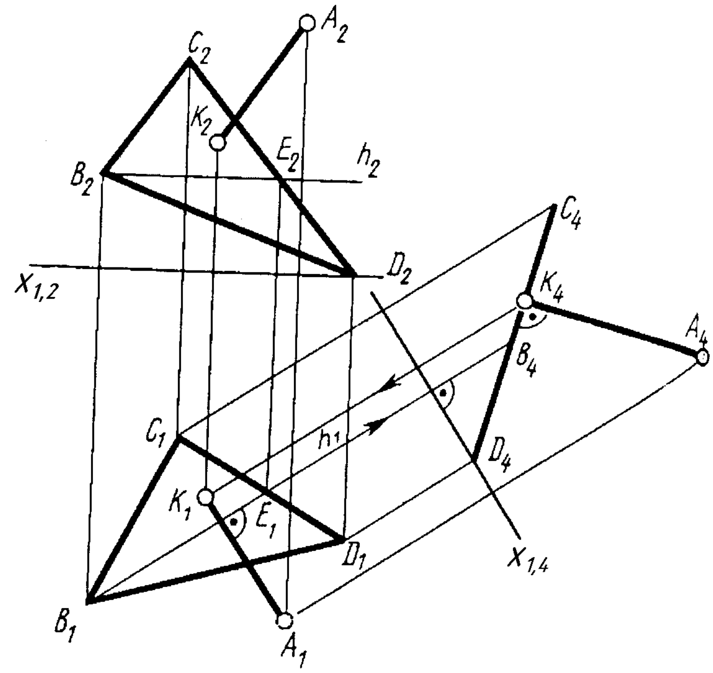


Fig.64

Problem 2. Determine the shortest distance between two crosslying straight lines (fig.65).

In the construction that fig.65 shows one of the crosslying straight lines (AB) is projected into a point onto additional projection plane Π_5. The construction has been made according to the following plan:

a) we move from system $\Pi_1 \perp \Pi_2$ to system $\Pi_1 \perp \Pi_4$, where $\Pi_5 \parallel AB$;

b) we move from system $\Pi_1 \perp \Pi_4$ to system $\Pi_4 \perp \Pi_5$, where $\Pi_5 \perp AB$;

c) having got on projection plane Π_5 a projection of straight line AB as a point a projection of another straight line C_5D_5, we will find the searched distance between two crosslying straight lines AB and CD.

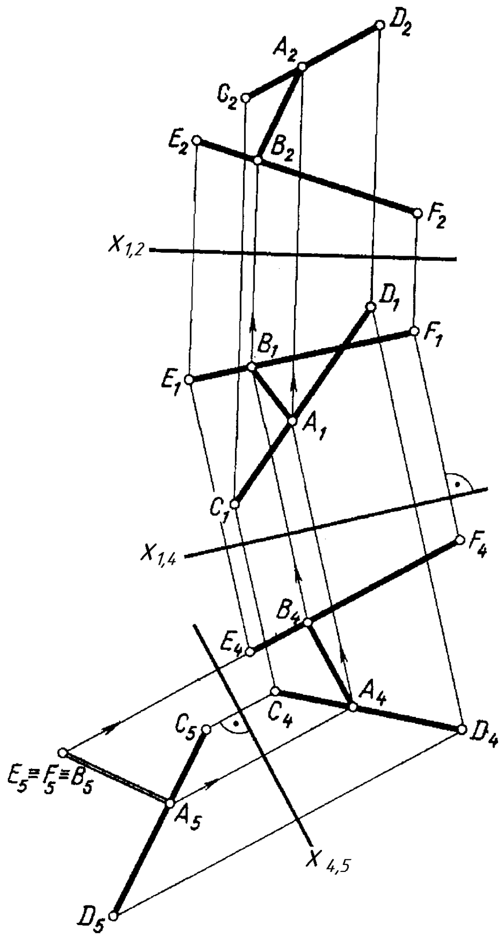


Fig.65

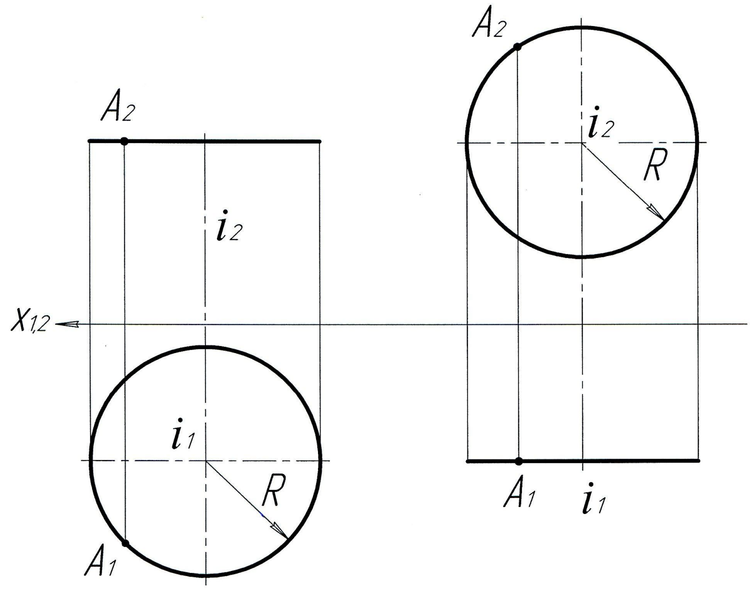
Questions to unit "A projection plane replacement method"

1. What is the essence of a projection plane replacement method?
2. Which condition should you follow while constructing a new projection plane?
3. How are projecting rays directed in relation to a new projection plane?
4. From which projection plane are the distances measured while moving to a new projection plane?
5. Describe the sequence of drawing new projection planes to determine a natural size of a straight line segment of a general position.
6. How should one make a general position segment a projecting one with the help of a projection plane replacement method?
8. Determine on your own the distance between a point and a general position segment by a projection plane replacement method.

Unit 7. A METHOD OF ROTATION AROUND THE AXIS,

PERPENDICULAR TO A PROJECTION PLANE

While rotating around some fixed straight line (rotation axis), each point of rotating figure moves in a plane that is perpendicular to a rotation axis (a rotaion plane). The point moves in a circle, the centre of which lies in an intersection point of the axis and the rotation plane (rotation centre) and the circle radius is equal to the distance from the rotation point to the centre (rotation radius). Let point A rotate around axis *i* that is perpendicular to Π_1 (fig.66a). We draw plane α through point A that is perpendicular to rotation axis *i* and that is parallel to plane Π_1.



а)　　　б)

Fig.66

While rotating point A depicts in plane α a circle of radius R that is equal to the perpendicular length from point A to the axis. The circle depicted in space by point A of radius $R=i_1A_1$ is projected onto plane Π_1 without deformation; on plane Π_2 this circle is depicted by a straight line segment, the length of which is equal to 2R.

Fig.66b shows rotation of point A around axis i that is perpendicular to Π_2. The circle which is depicted by point A is projected without deformation onto plane Π_2. A circle of radius $R=i_2A_2$ is drawn from point i_2 as from the centre; on plane Π_1 this circle is depicted by a straight line segment, the length of which is equal to 2R.

Problem. Make straight line AB of a general position a horizontal projecting one by gradual rotation around the axes that are perpendicular to the projection planes (fig.67).

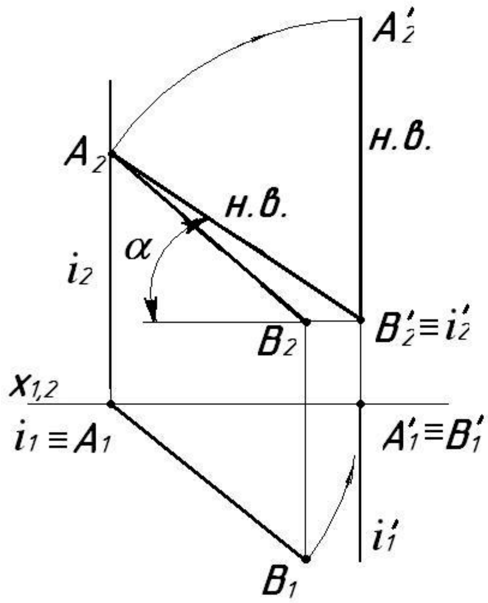


Fig.67

Solving. We draw rotation axes so, that they intersect straight line AB. Due to it the construction is simplified, as a straight line point which lies on axis will be constant, and that is why to determine the rotated position of a straight line we should rotate its only one point.

At first we rotate straight line AB around a vertical axis to a front line position. To do this, it is enough to rotate point B_1 around centre i_1 to the position of B_1 so to make rotated projection A_1B_1 perpendicular to link line A_1A_2, and then to find a front projection B_2 of point B. We connect points A_2 and B_2. Straight line AB has become parallel to plane Π_2, so segment A_2B_2 is equal to a natural size of segment AB, angle α is equal to an inclination angle of straight line AB to plane Π_1. By the second rotation around axis O which is perpendicular to Π_2 we put straight line AB into position A_2B_2 athwart plane Π_1. A horizontal projection of straight line AB is projected into a point on Π_1 (A_1=B_1).

Questions to unit "A method of rotation around the axis, perpendicular to a projection plane"

1. What is the essence of a method of rotation around the axis, perpendicular to a projection plane?
2. Which one of the projections doesn't change its size while rotating?
3. How do the points move on the opposite plane?
4. Determine on your own a natural size of a general position segment by a rotation method.

Unit 8. A PLANAR MOVEMENT METHOD (A ROTATION AXIS IS NOT MENTIONED)

The essence of this method is that projection planes remain constant and figures (a point, a line, a plane) in space are moved into a desirable position. Herewith, one of the projections of the specified figure doesn't change its shape and size while being moved into the necessary position. A rotation axis is not mentioned.

The use of a planar movement method is based on solving of four main problems.

Problem 1. Move segment AB of a straight line of a general position so, that it becomes parallel to plane Π_2 (fig.68).

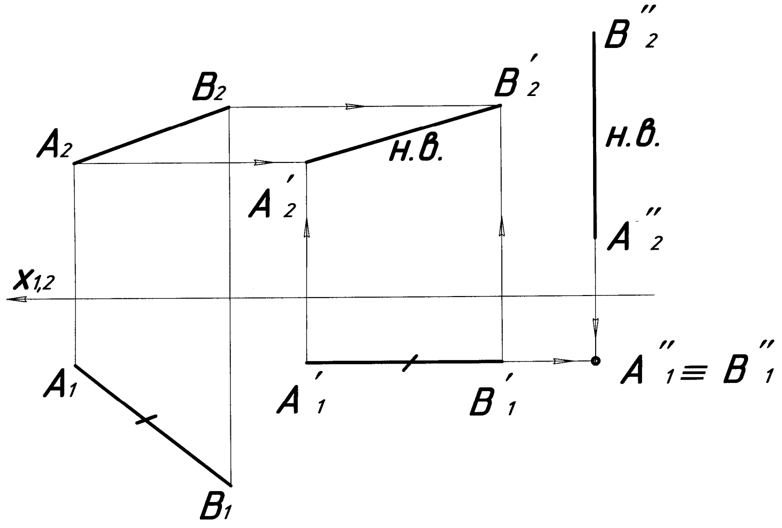


Fig.68

Solving. We take horizontal projection A_1B_1 and move it parallel to axis X_{12} which corresponds to the parallel position of the segment itself to Π_2. Herewith, $A_1B_1=\bar{A}_1B_1$.

To get a front projection of points A and B, we draw link lines athwart axis X_{12} and from projections of points A_2B_2 – link lines parallel to axis X_{12}. At the intersection of these link lines we will get projections of points A_2B_2, i.e. a natural size of straight line AB. Here we can mark inclination angle α of this straight line to horizontal projection plane Π_1.

Problem 2. Move segment $AB \perp \Pi_1$.

Solving. First we solve problem 1. Then we place projection $\bar{A}_2B_2 \perp X_{12}$, taking into consideration that $\bar{\bar{A}}_2\bar{\bar{B}}_2 = \bar{A}_2\bar{B}_2$ (fig. 68). At the intersection of the link lines from A_2B_2 athwart axis X_{12} and from A_1B_1 parallel to axis X_{12} we shall get $A_1 \equiv B_1$, i.e. segment $AB \perp \Pi_1$.

By the first movement a point projection has the same mark with one line above a letter, by the second movement – two lines above a letter. The constructions that are made correspond to rotations around the axes that are perpendicular to the projection planes, but these axes are not shown.

Problem 3. Move a general position plane that is specified by triangle ABC into a front projecting position (fig.69).

Solving. We should take a horizontal line (h_2h_1) in a plane of triangle ABC and move it into a position athwart Π_2. Then a triangle itself, which this horizontal line belongs to, will become perpendicular to Π_2. As we make the construction without mentioning the rotation axes, we place projection $A_1B_1C_1$ at random, but so that a horizontal line will become perpendicular to X_{12}. We mark A_1 and l_1 on a horizontal line, keeping distance A_1l_1. We get a new position of points B_1 and C_1 with the help of the compasses by putting marks. Herewith, a horizontal projection of a triangle keeps its shape and size ($\overline{\overline{A}}_1\overline{\overline{B}}_1\overline{\overline{C}}_1 = \overline{A}_1\overline{B}_1\overline{C}_1$), only its position changes. At the intersection of the link lines from points A_1, B_1, C_1 athwart axis X and link lines from points A_2, B_2, C_2 parallel to axis X we shall get a front projection of a triangle in a shape of a straight line, i.e. of a front projection position ($A_2B_2C_2 \perp \Pi_2$). We can also mark angle α here – an inclination angle of this plane to a horizontal projection plane.

Problem 4. Move a general position plane specified by triangle ABC into a position that is parallel to a horizontal projection plane (fig.69).

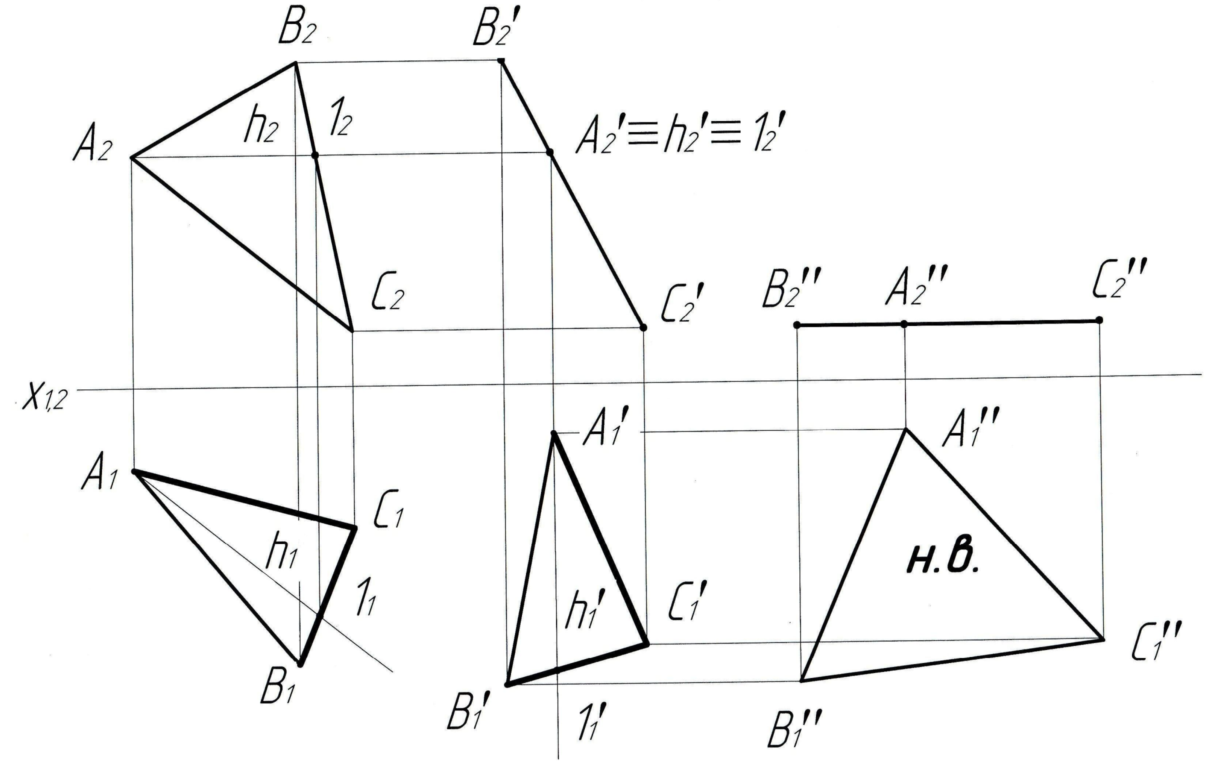


Fig.69

Solving. To get such a triangle position we should first solve problem 3. Then we move a front projection of a triangle in a shape of straight line $A_2B_2C_2$ along axis X and parallel to it, so that projection $A_2B_2C_2$ keeps its shape and size that have been got while solving problem 3($\overline{\overline{A}}_2\overline{\overline{B}}_2\overline{\overline{C}}_2 = \overline{A}_2\overline{B}_2\overline{C}_2$). We get a horizontal projection of a triangle at the intersection of the link lines from A_2, B_2, C_2 athwart axis X. Projection $A_1B_1C_1$ gives a natural size of triangle ABC.

Due to a planar movement method one determines a distance from a point to a plane, specified by different methods; a distance between two parallel and crosslying straight lines etc.

Let's examine a problem to determine a two-facet angle with edge AD (fig.70).

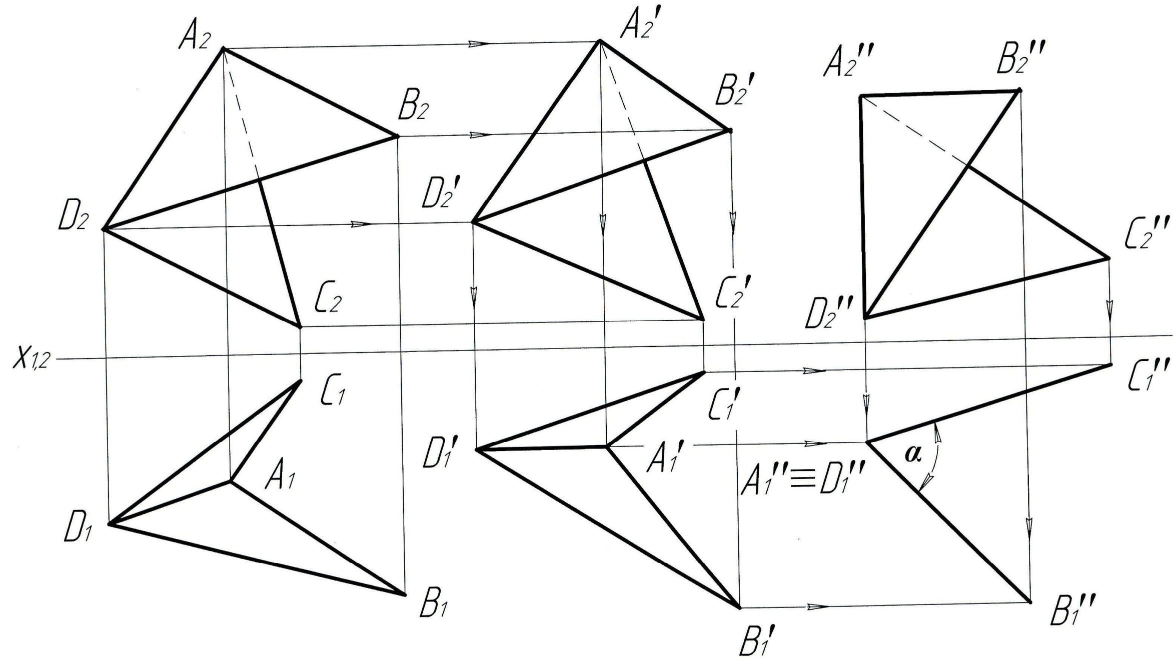


Fig.70

The basis of this problem is formed by problems 1 and 2, i.e. a two-facet angle with edge AD is projected into a natural size, if edge AD is projected into a point and the side facets – into the straight lines.

We move a horizontal projection of a figure along the axis so, that edge AD becomes parallel to axis X and so, that $A_1D_1 = \overline{A}_1\overline{D}_1$.

We transfer points B and C with the help of the compasses by putting marks. The moved figure should not change the shape and the size of the specified one.

We shall get a front projection of the two-facet angle at the intersection of the link lines, the directions of which are pointed by arrows.

By the second movement we move a front projection of the two-facet angle so, that AD becomes perpendicular to axis X. Then we move points B_2 and C_2 with the help of the compasses by putting marks. We get a horizontal projection of an angle with the help of the link lines. Points A and D coincide into one point and facets ADB and ADC – into straight lines. Angle α determines a natural size of an angle with edge AD.

Questions to unit "A planar movement method"

1. What is the essence of a planar movement method?
2. What main problems can be solved with the help of this method?

3. In what sequence is the movement made while determining a natural size of a general position triangle?

4. How can one find a natural size of a two-facet angle with the help of a planar movement method?

Unit 9. A METHOD OF ROTATION AROUND THE AXIS, PARALLEL TO A PROJECTION PLANE

Fig.71 shows a segment of straight line AB of a general position. We will draw straight line *i* parallel to plane Π_1 that intersects segment AB in point K. Taking straight line *i* for a rotation axis, we shall rotate segment AB around it so, that it becomes parallel to Π_1. In a rotated position of segment AB its front projection A_2B_2 coincides with front projection i_2 of rotation axis *i*, and horizontal projection A_1B_1 will determine a natural size of segment AB.

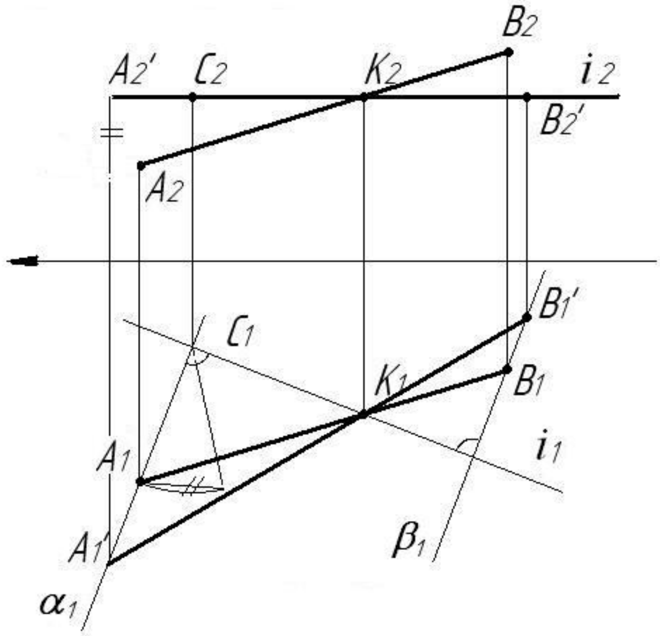


Fig.71

One should make the construction of horizontal projection A_1B_1 of a segment of a rotated position the following way. While rotating around axis *i*, points A and B will move in horizontal projecting planes α and β that are perpendicular to rotation axis *i*. Therefore, projections A_1 and B_1 of the ends of segment AB in its new position AB will lie on the traces according to α_1 and β_1 of these planes. A rotation radius of points A and B is projected onto plane Π_1 by a horizontal position of segment AB into its natural size. With the help of a right-angled triangle we find a natural size of radius r_a of point A and put r_a from point

C_1 (a rotation centre of point A) on trace a_1. Having connected obtained point A_1 with projection K_1 of constant point K of the intersection of axis *i* and straight line AB, we shall find a horizontal projection of straight line AB after rotation of AB around axis *i.* At the intersection of projection A_1K_1 and trace β_1 we have horizontal projection B_1 of point B. Projection A_1B_1 is equal to a natural size of segment AB.

Fig.72 shows the construction of a natural size of a plane figure with the help of a method of rotation around the axis, parallel to a projection plane.

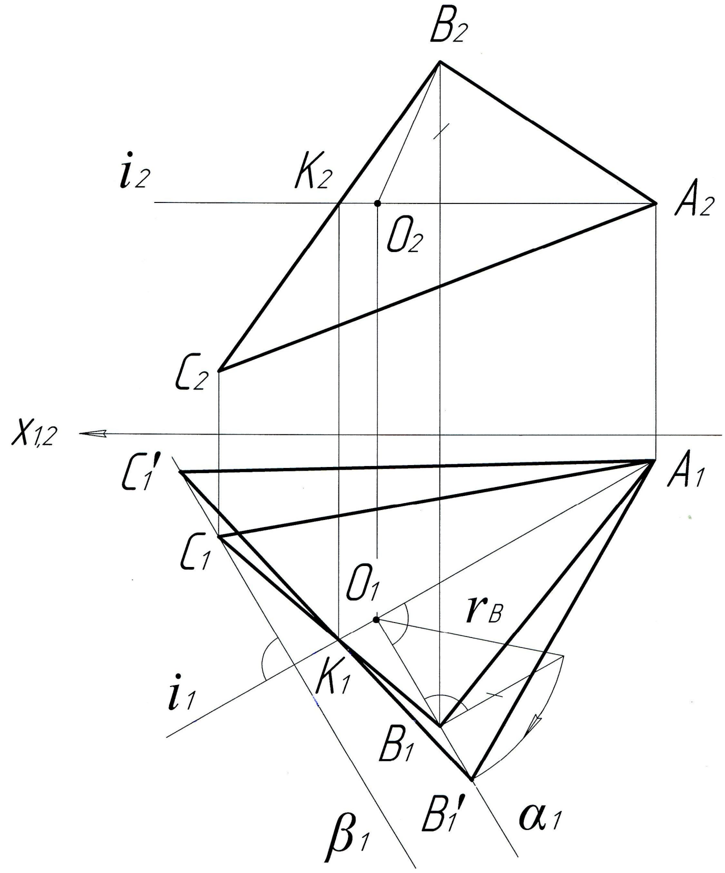


Fig.72

With the help of this method triangle ABC is put into the position that is parallel to plane Π_1, after that it will be projected into its natural size onto plane Π_1. Front projection $A_2B_2C_2$ of triangle ABC after rotation around axis *i* has coincided with a front projection of axis i_2. To construct triangle $A_1B_1C_1$ we put down a perpendicular from B_1 onto projection i_1 of rotation axis *i*. With the help of a right-angled triangle we find a natural size of radius r_b of rotation of point B and move it onto the drawn perpendicular (the trace of plane α). Point B_1' is a projection of vertex B of the given triangle in its position that is parallel to plane Π_1.

Having drawn a straight line through points B_1' and K_1 to the intersection with a perpendicular that is put down from C_1 onto i_1 (by trace of plane β), we shall find point C_1 that will be a horizontal projection of vertex C of triangle ABC in its position that is parallel to plane Π_1. Vertex A of the triangle is constant as a point that lies on the rotation axis. Having connected its projection A_1 and projections B_1' and C_1' by the straight lines, we shall find horizontal projection $A_1B_1C_1$ of triangle ABC that is parallel to plane Π_1, i.e. we shall find a natural size of triangle ABC.

Questions to unit "A method of rotation around the axis, parallel to a projection plane"

1. What is the essence of a method of rotation around the axis, parallel to a projection plane?
2. How does one change a position of a point projection by rotation around the axis that is parallel to plane Π_1?
3. How can one find a natural size of a triangle of a general position with the help of the given method?

Unit 10. CURVES

It is important to examine curves as generating lines of curved surfaces in descriptive geometry. A curve can be made up by the movement of a point in space, by the intersection of curved surfaces and a plane, by the mutual intersection of two planes. There are plane and space curves.

Plane curves are curves, all the points of which lie in one plane. Space curves are curves, all the points of which don't belong to one plane.

Questions to unit "Curves"

1. How are curves made up?
2. Which curves are called plane curves?
3. Which curves are called space curves?

Unit 11. SURFACES. CLASSIFICATION OF SURFACES

A surface is a geometric place of the consistent positions of lines (generating lines), that are moved in space according to a certain law (a guiding line).

Such an image of the surface is kinematic. A surface that is constructed due a law of the generating lines movement is called a logical surface, unlike an accidental surface.

On any kinematic surface there are two families of lines: generating lines and guiding lines that can exchange their roles. Therefore, one surface can be made up by the movement of different lines. If a generating line of a surface is a straight line, a surface is called a rectilinear surface. If a generating line of a surface is a curve, a surface is called a curvilinear surface.

Surfaces that can exactly coincide with the graphic plane without folds and breaks are called unfolded surfaces.

The unfolded surfaces can only be those surfaces, on which two infinitely close positions of the generating lines are either parallel to each other, or they intersect. A cylinder, a cone and a torso belong to the unfolded surfaces. The rest of the surfaces are the folded surfaces. Fig.73 shows classification of surfaces.

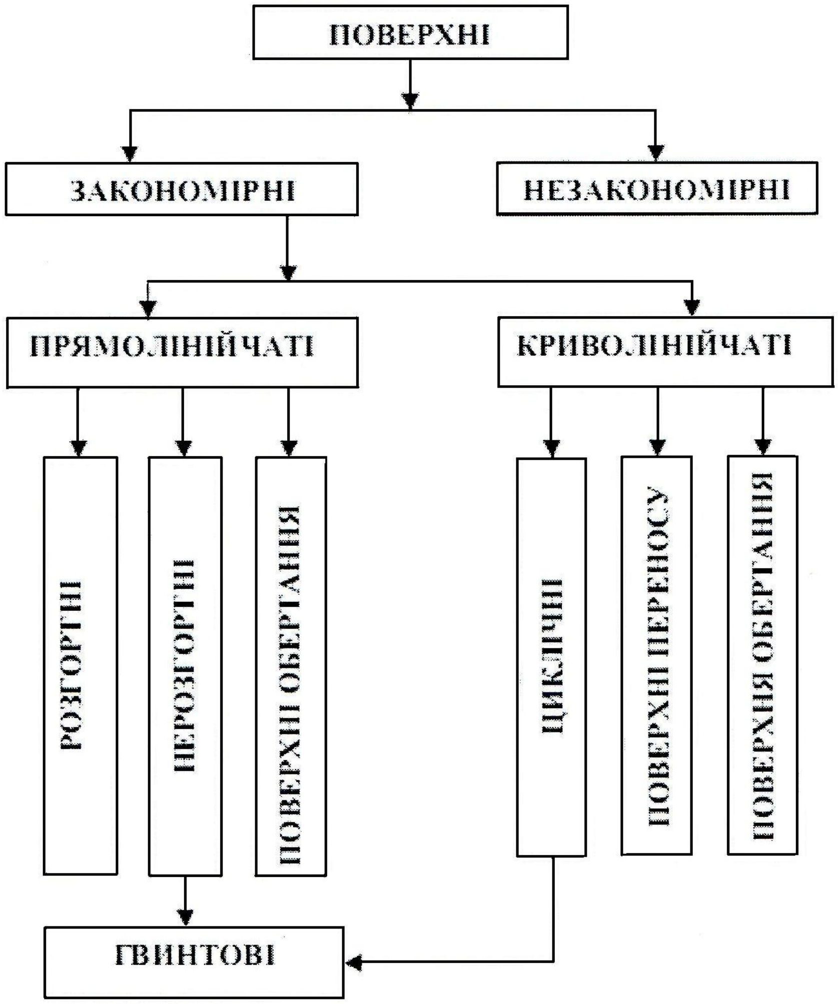


Fig.73

11.1. A surface determinant

The set of the basic parameters of a surface is called a surface determinant. A surface determinant consists of two parts. The first part is a geometric part of a determinant (GPD). This is a list of all the geometric elements that take part in the construction of the given surface.

The second part is an algorithmic part of a determinant (APD), i.e. an algorithm of the surface formation from the geometric elements that are included to the composition of a determinant.

We mark a surface determinant with a letter ***Φ***. Let us write down a determinant of a cylindrical surface:

GPD *Φ* (ã; ē; s;) →

APD *li* × ã *li;* ‖ *S*,

where **ã** is a guiding curved line; ***l*** is a generating line; ***S*** is a specified direction.

APD shows that any generating line of a cylinder (in any position) should intersect a guiding curved line **ã** and it remains parallel to the specified direction ***S***.

11.2. A cylindrical surface

By construction of a cylindrical surface a generating line ***l*** in any position should intersect a guiding line **ã** and should be parallel to the specified direction ***S*** (S_2, S_1) (fig.74).

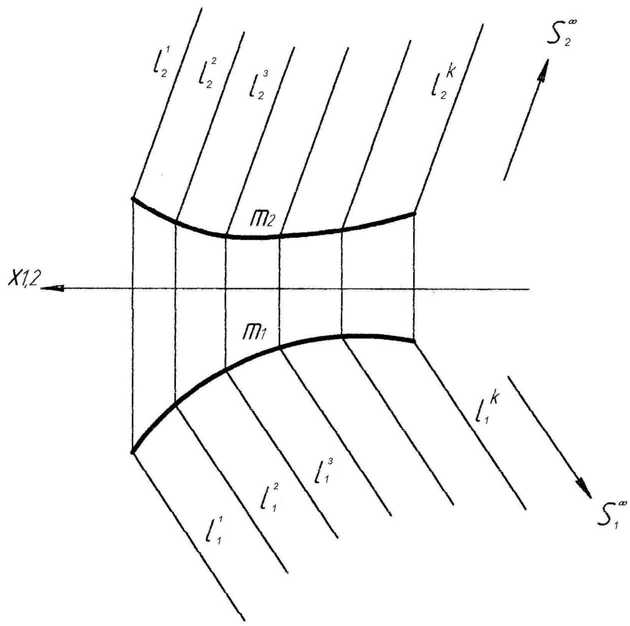


Fig.74

A projection of a point that belongs to the surface lies on a projection of a generating line ***l***. Fig. 74 shows that a projection of point A lies on a projection of generating line ***l'''***.

11.3. A conic surface

Each surface is constructed by the movement of generating line ***l*** that goes through one constant point ***S*** – a vertex of a cone (fig.75).

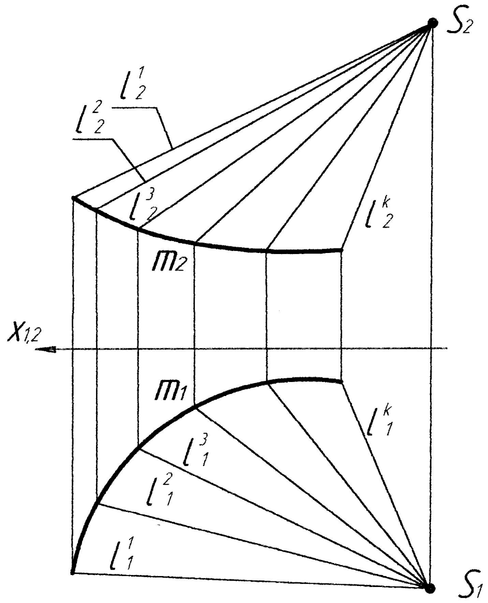


Fig.75

11.4. A surface with a curved edge (a torso)

A curved edge can be any guiding curve, if it is spatial. A torso surface is constructed, when in any point of this guiding line a straight line (a generating line) will be tangent (fig.76).

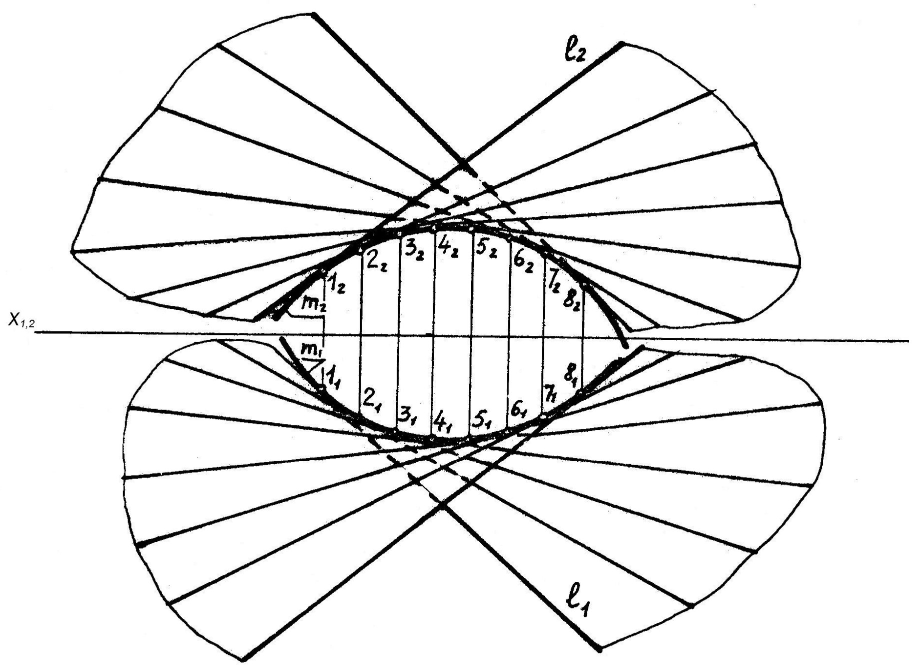


Fig.76

11.5. Rectilinear folded surfaces (with a plane of parallelism or Catalan surface)

These are the surfaces, in composition of which two guiding lines and a plane take part, and a generating line of a surface in any position is parallel to the plane. Such a plane is called a plane of parallelism. A plane of parallelism can be any plane in the system of projection planes or it can be a projection plane itself.

A Belgian scientist Catalan did a research concerning the surfaces with a plane of parallelism, that is why these surfaces are called the surfaces with a plane of parallelism or Catalan surfaces. Such surfaces include a cylindroid, a conoid, an oblique plane (a hyperbolic paraboloid).

A surface that has two guiding lines that are curves and a plane of parallelism is called a cylindroid (fig.77).

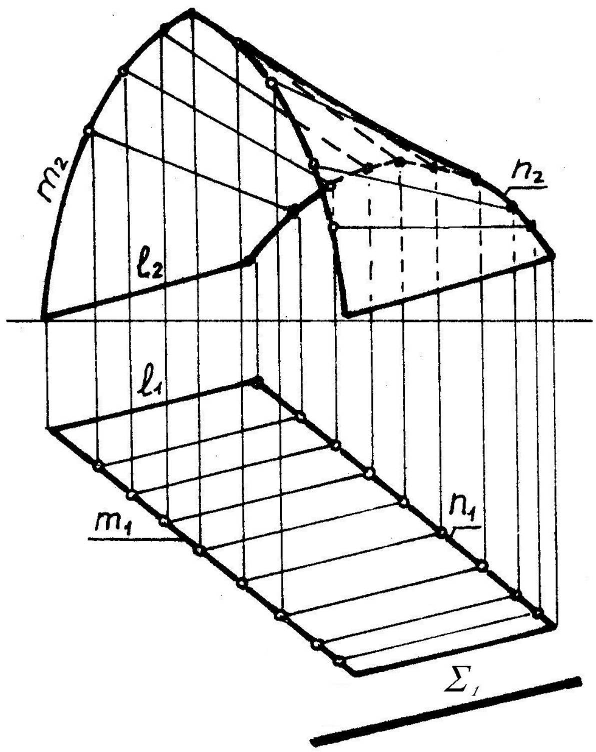


Fig.77

A surface that has one guiding line which is a straight line and another guiding line which is a curve and a plane of parallelism is called a conoid (fig.78).

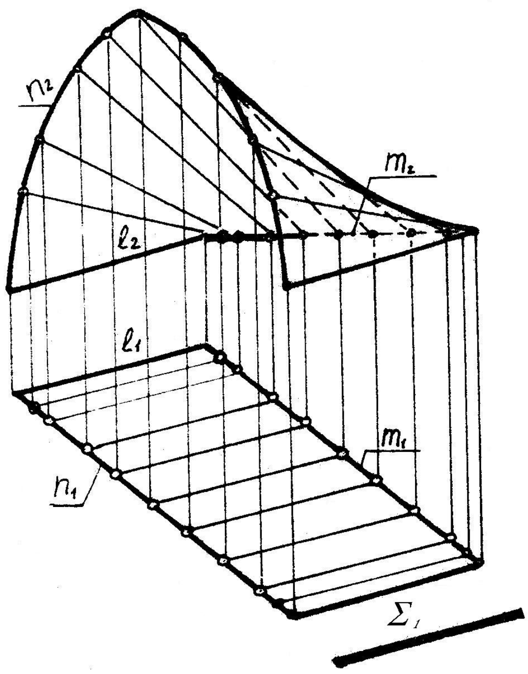


Fig.78

A surface that has two guiding lines which are the straight lines and a plane of parallelism is called an oblique plane or a hyperbolic paraboloid (fig.79).

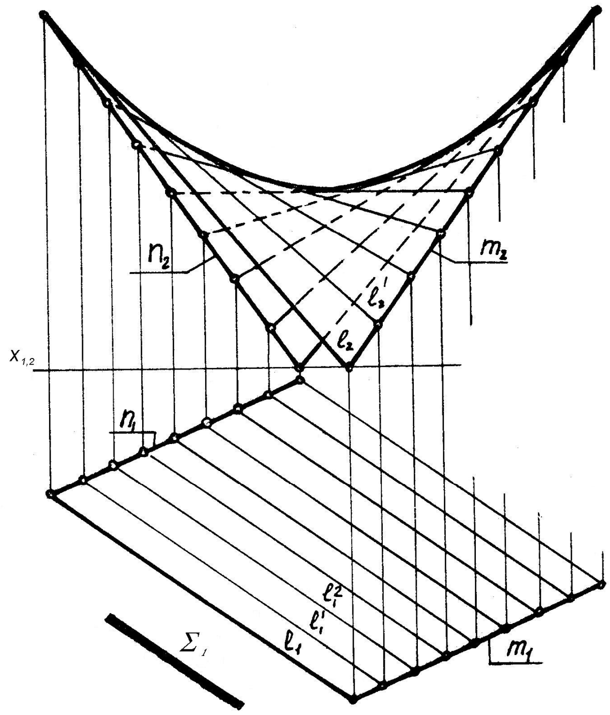


Fig.79

11.6. Rectilinear surfaces of rotation

A rectilinear surface of rotation is a surface that is made up by rotation of a generating line (a straight line) around a constant axis.

Let us study three cases:

1. Generating line ***l*** and axis ***i*** intersect – that is a circular cone (fig.80).

2. Generating line ***l*** is parallel to a rotation axis – that is a circular cylinder (fig.81).

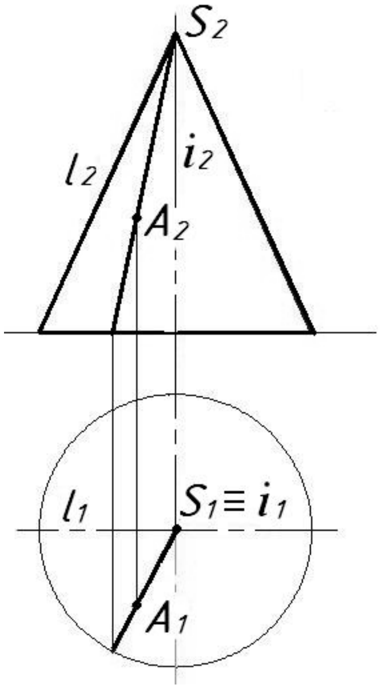


Fig.80

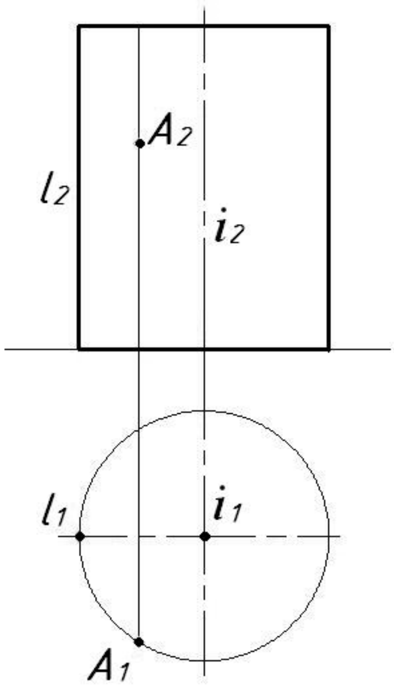


Fig.81

3. Generating line ***l*** is crosslying to rotation axis ***i*** – that is a hyperboloid of rotation of one sheet (fig.82).

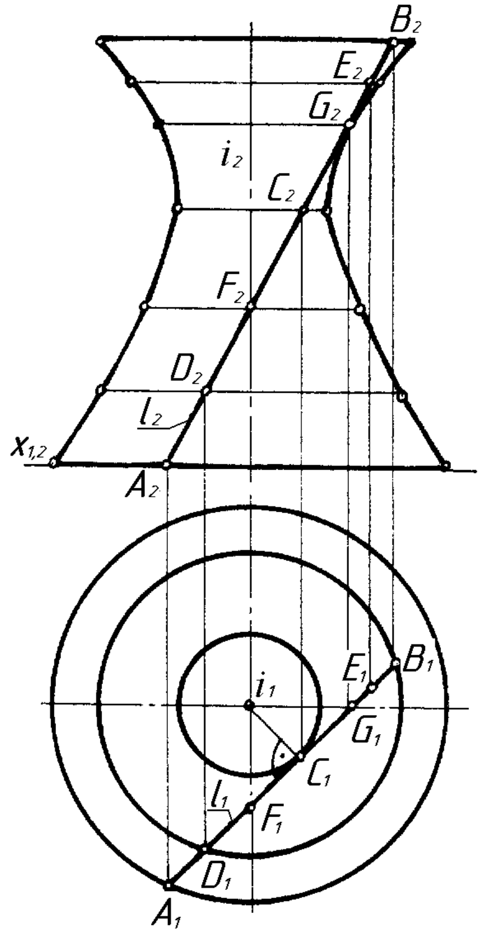


Fig.82

11.7. Curvilinear surfaces of rotation

These are the surfaces which are made up by rotation of a generating line (a curve) around a constant axis. A generating curved line can be both a plane curve and a space curve (fig.84).

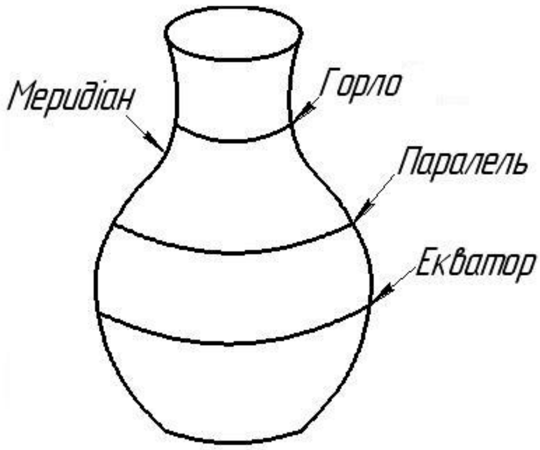


Fig.83

By rotation of a generating line each point on this line describes a circle with a centre on rotation axis ***i***. These circles are called parallels.

A parallel, a diameter of which is bigger than a diameter of other parallels is called an equator. A parallel, a diameter of which is smaller than the diameters of other parallels is called a neck.

In a general case a surface of rotation can have several equators and necks. Planes α that go through a rotation axis are called meridional planes and lines, on which they intersect a surface are called meridians.

Meridional surface α that is parallel to a projection plane is called a main meridional plane and its intersection line with a surface of rotation is called a main meridian.

The projections of a surface of rotation and the construction of a point projection on this surface is shown on fig.84.

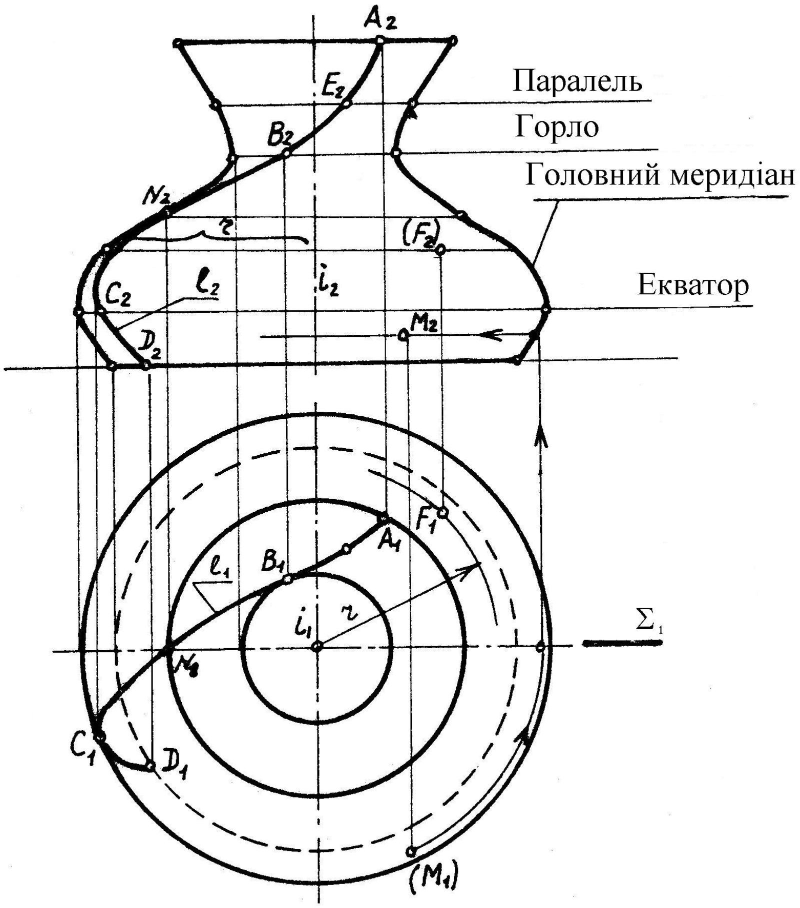


Fig.84

Let’s examine some surfaces of rotation:

1. A sphere.

A sphere surface is made up by rotation of a circle around its diameter (fig.85).

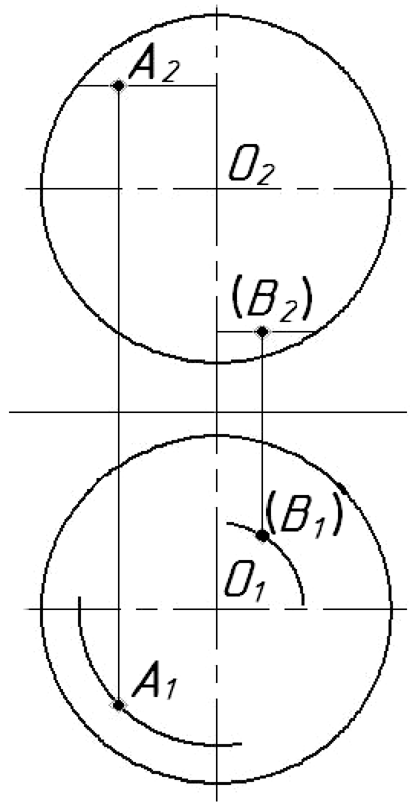


Fig.85

2. A torus.

A torus surface is made up by rotation of a generating circle around axis ***i*** (fig.86). There are two types of a torus that are known:

a) an open torus, when a generating circle doesn’t intersect a rotation axis;

b) a closed torus, when a generating circle intersects a rotation axis.

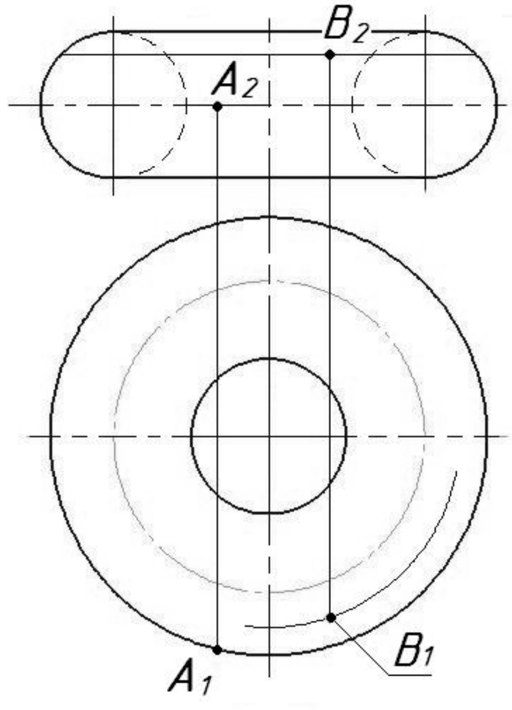


Fig.86

3. An ellipsoid of rotation.

A surface of an ellipsoid of rotation is made up by rotation of an ellipse around its axis (fig.87).

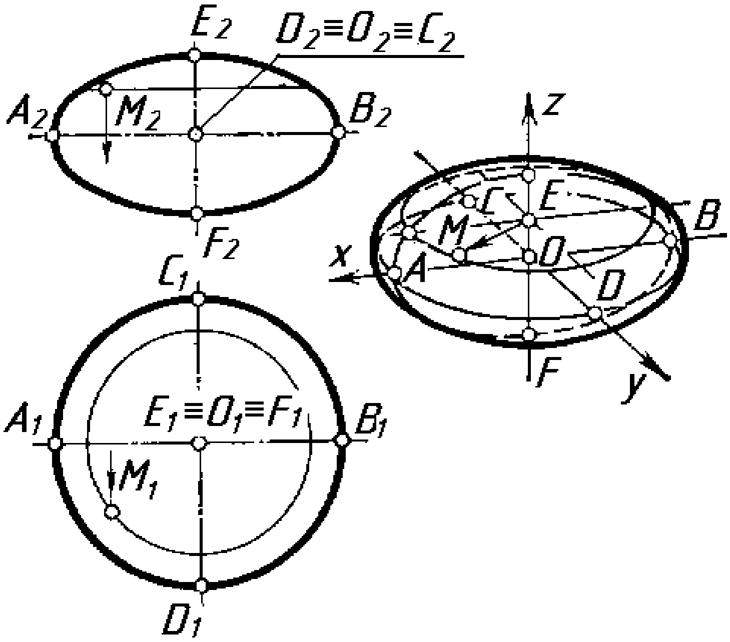


Fig.87

4. A paraboloid of rotation.

A surface of a paraboloid of rotation is made up by rotation of a parabola around its axis (fig.88).

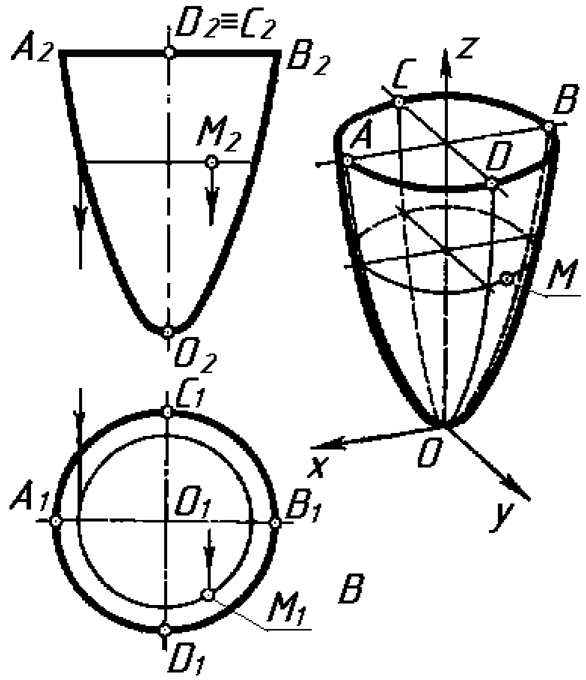


Fig.88

5. A hyperboloid of rotation.

A hyperboloid of rotation of one sheet is made up by rotation of a hyperbola around its imaginary axis (fig.89). A hyperboloid of rotation of two sheets is made up by rotation of a hyperbola around its real axis (fig.90).

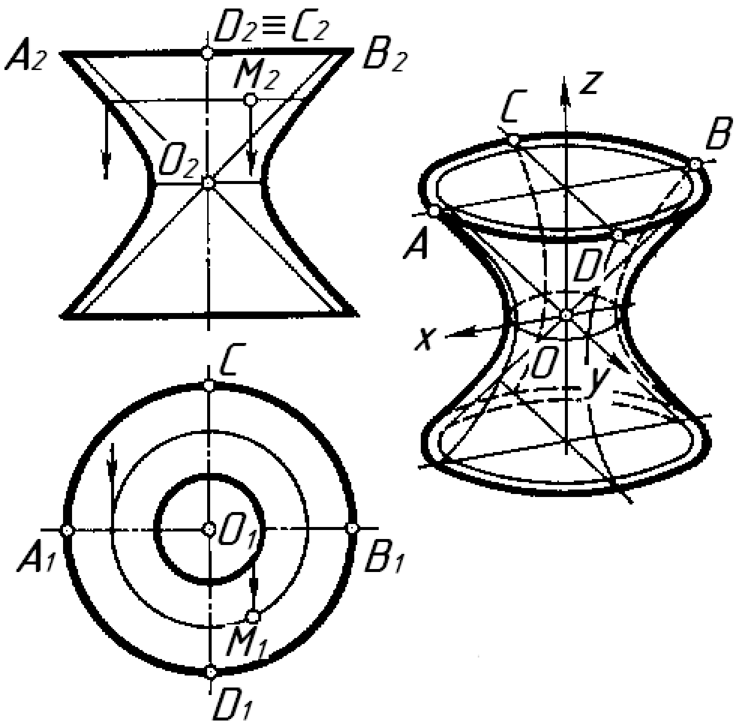


Fig.89

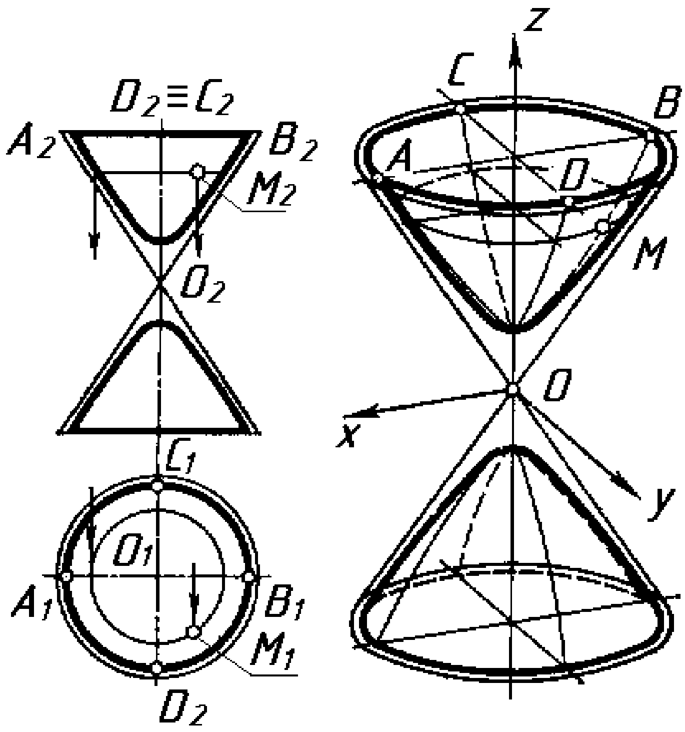


Fig.90

11.8. Circular surfaces

Circular surfaces are the surfaces that are made up by the movement of a circle of a constant or a variable radius along the guiding line that goes through the circle centre (fig.91).

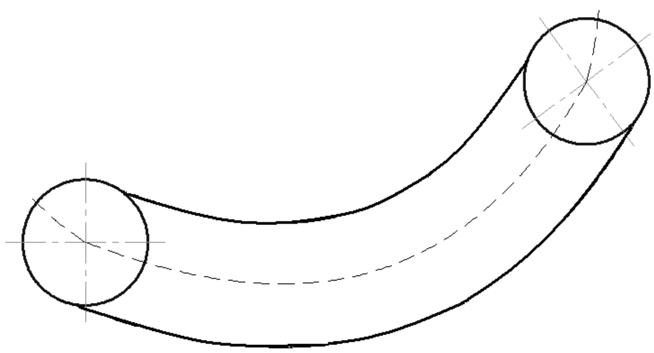


Fig.91

Circular surfaces include the channel surfaces and the tubular surfaces. The channel surface is made up by the movement of a circle of a variable radius along the guiding curved line, herewith, an area of a circle in any position is perpendicular to a guiding line.

The tubular surface differs from the channel surface by the fact, that a radius of a generating circle or of a generating sphere is constant (fig.91).

11.9. Moving surfaces

A moving surface is made up by a continuous progressive movement of a generating curved line, which remains parallel to the initial position in every new position (fig.92).

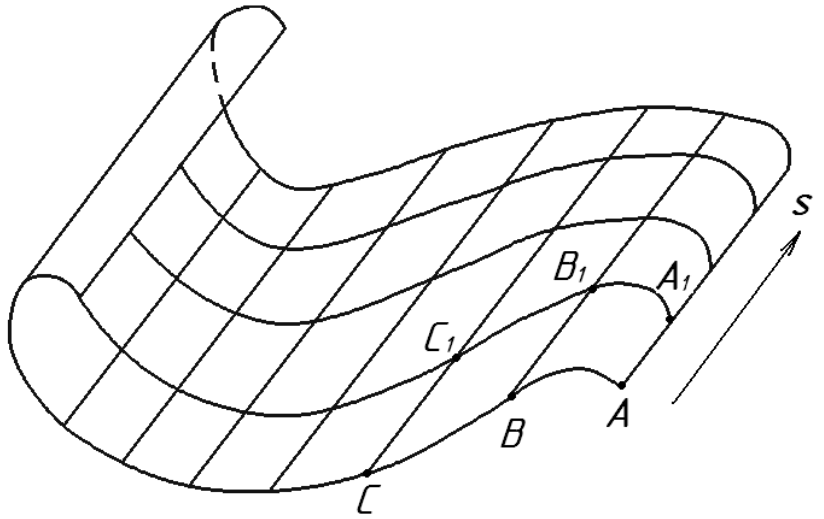


Fig.92

A moving surface is specified by a start position of generating line ABC and by the direction of movement on fig.92. Curves of ABC, $A_1B_1C_1$, … are a number of positions of a generating line and they determine a mesh of a moving surface.

11.10. Helix surfaces

A surface is called a helix surface, if it is formed by a helix movement of a generating line.

A helix movement is characterized by rotation around the axis and at the same time by motion that is parallel to this axis.

If a generating line is a straight line in a helix surface, a surface is called a helicoid. Depending on an angle of inclination of a generating line to a helix axis, a helicoid is called a right one, if this angle is equal to 90° and an oblique one, if this angle is not equal to 90°. In their turn, the right and oblique helicoids are divided into the closed and open helicoids.

They are called the closed helicoids when a generating line intersects an axis of a helix surface. They are called the open helicoids when a generating line and an axis of a helix surface are crosslying.

The right, oblique, closed and open helicoids can be the ring helicoids, if a coaxial cylinder intersects this helix surface.

Fig.93 shows the projections of a closed right helicoid.

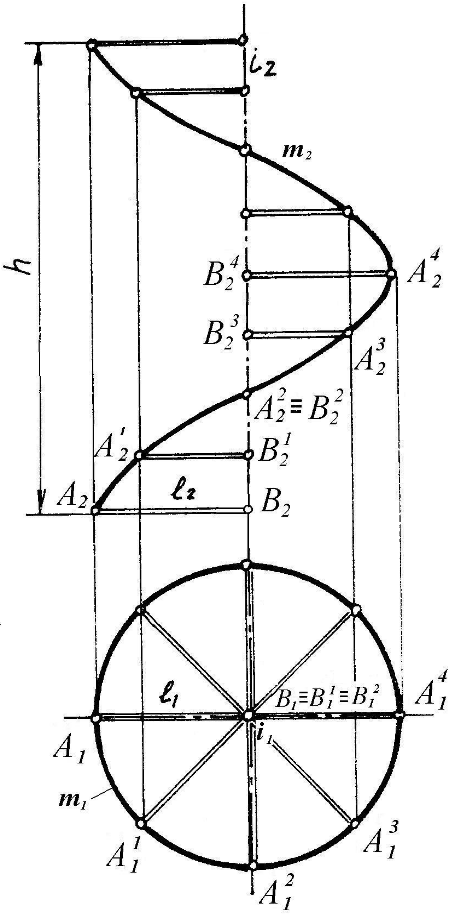


Fig.93

Questions to unit "Surfaces. Classification of surfaces"

1. What is called a surface?
2. Which surfaces are called the logical surfaces? Give your examples.
3. Which surfaces are called the rectilinear surfaces? Give your examples.
4. Which surfaces are called the curvilinear surfaces? Give your examples.
5. Which surfaces are called the unfolded surfaces? Give your examples.
6. What is called a surface determinant? What parts does it consist of?
7. What is called a curved edge?
8. Which plane is called a plane of parallelism?
9. Name the surfaces that have a plane of parallelism.
10. Which surfaces are called the helix surfaces?

Unit 12. INTERSECTION OF THE EDGED AND CURVED SURFACES AND A PLANE. EVOLVENT. DETERMINATION OF A NATURAL SIZE OF A SECTION

At the intersection of the surfaces and a plane we shall get a plane figure that is called a section and it belongs to an intersecting plane. They usually start to determine a projection of a section line with the construction of the anchor points – the points that lie on the extreme contour generating lines of the surface, the highest and the lowest points of the figure, points that determine the visibility limit. After that they determine the arbitrary points of a section figure.

12.1. Construction of a surface section by a projecting plane. A natural size of a section. Evolvent

Problem. Construct an intersection line of a sphere and a front projecting plane α (fig.94).

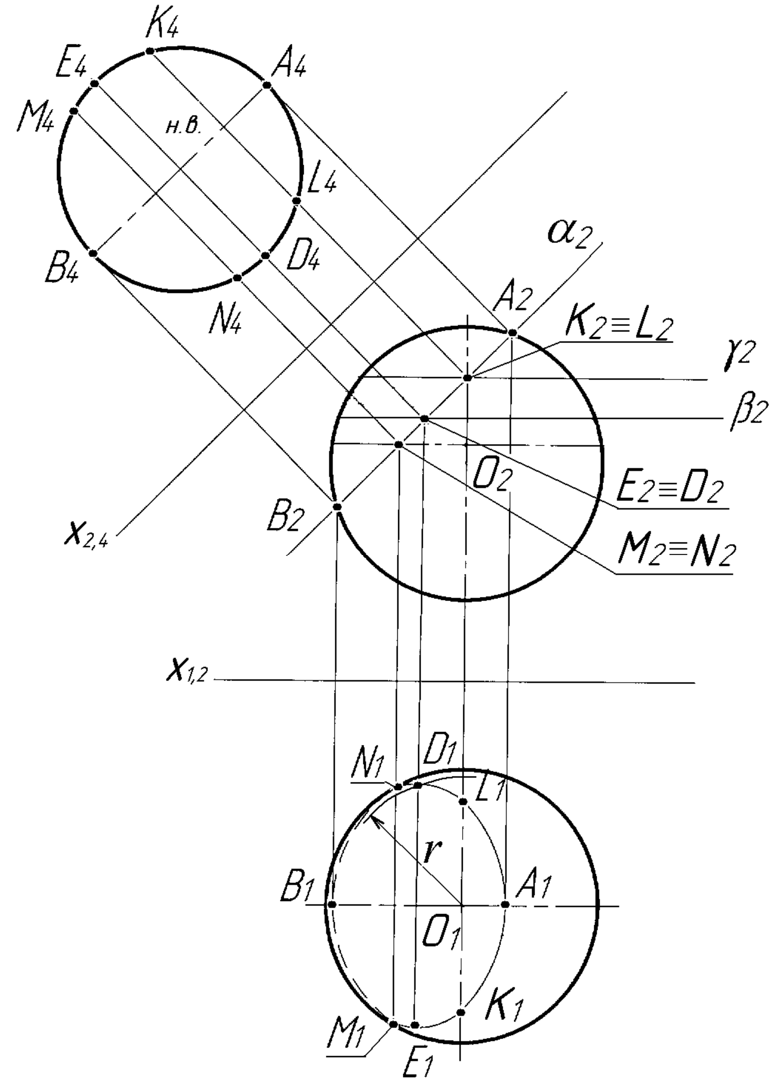


Fig.94

A plane intersects a sphere in circle. We have already a front projection of this circle as a projection that coincides with the projection of an intersecting plane. Now we have to construct a horizontal projection. This will be an ellipse. At first we construct the projections of anchor points. The highest point of a section figure is point A (A_1, A_2), the lowest point is point B (B_1, B_2). On the equator L

(L_1, L_2) of a sphere we mark points M (M_1, M_2) and N (N_1, N_2) which are the visibility points. These points divide a horizontal projection of a curve into two parts – a visible one and an invisible one. We find the axes of the ellipse, in which a circle of this section is projected onto plane Π_1. A small axis A_1B_1 of the ellipse coincides with a horizontal projection of the main meridian of a sphere.

Projection E_2D_2 of a big axis of the ellipse of a section onto plane Π_2 is a point that lies in the middle of segment A_2B_2. We draw auxiliary horizontal plane β so, that its front trace β_2 goes through point $E_2 \equiv D_2$. This plane intersects the sphere in circle of radius r. From point C_1 as from a centre we draw a circle of radius r, which intersects a link line, drawn through points E_2D_2 in points E_1 and D_1. Segment E_1D_1 is a big axis of the ellipse. Other section points can be constructed with the help of the auxiliary horizontal planes. Thus, with the help of plane γ we shall find points K (K_1, K_2) and L (L_1, L_2).

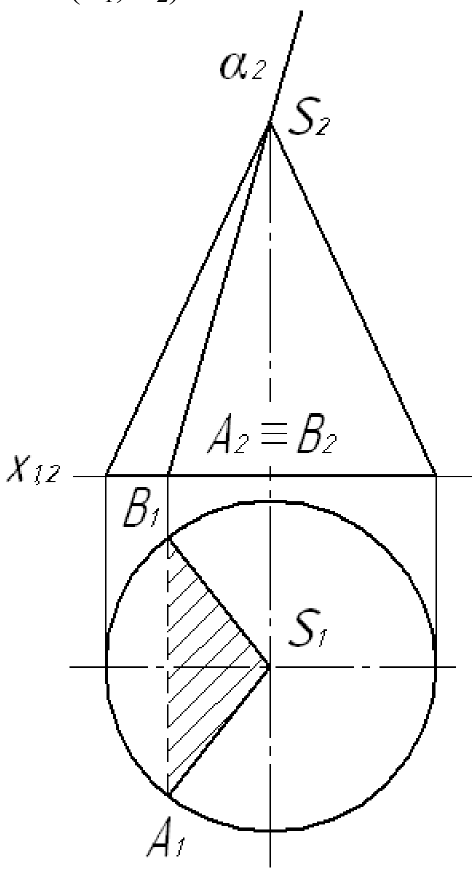


Fig.95

12.2. Conic sections

One can get the following lines on the surface of a right circular cone at the intersection by a plane:

1) two generating lines, if an intersecting plane goes through a cone vertex (fig.95, plane α);

2) a circle, if an intersecting plane is perpendicular to a cone axis (fig.96, plane β);

3) a hyperbola, if an intersecting plane is parallel to two arbitrary generating lines of a cone or if this plane is parallel to a cone axis (fig.97, plane α);

4) a parabola, if an intersecting plane is parallel to one of the generating lines of a cone (fig.98, plane α);

5) an ellipse, if a plane intersects an axis and the generating lines of a cone and if it is not perpendicular to a cone axis (fig.99, plane α).

Problem 1. Fig.97 shows a section of a cone by front plane α that doesn't go through a cone vertex. In this case on a lateral surface of a cone we shall get a hyperbola that is projected onto plane Π_1 into a straight line that is parallel to two generating lines of a cone, and that is projected onto plane Π_2 into its natural size. Points K and L of a hyperbola, in which it intersects plane Π_1 are determined by the intersection of a circle of a cone base and a trace of intersecting plane α. Front projections K_2 and L_2 of these points will lie on axis OX.

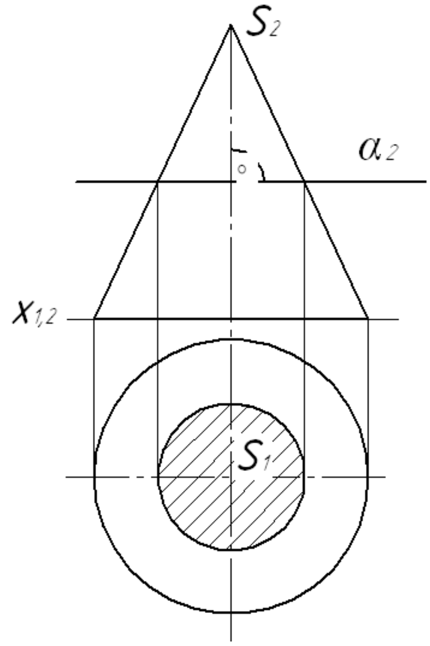


Fig.96

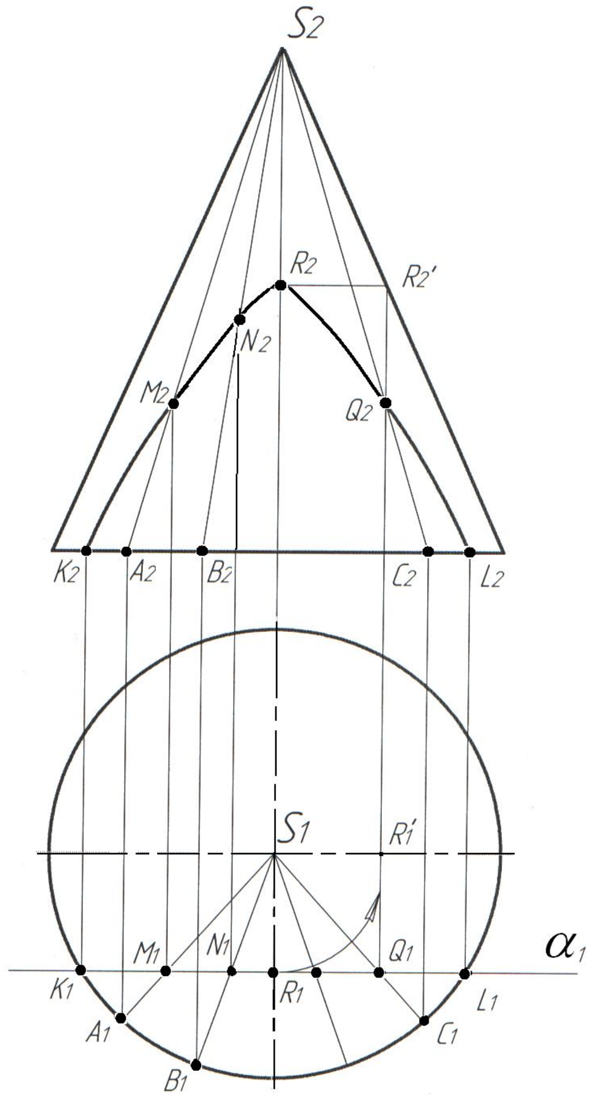


Fig.97

To construct a front projection R_2 of anchor point R that is a hyperbola top, we draw from point S_1 as from a centre a circle, the radius of which is equal to a distance from point S_1 to trace α_1. This circle is a horizontal projection of a cone section by a horizontal plane that goes through point R.

To find a front projection of this circle we draw a link line through R_1 to the intersection with a front projection of a right generating line of a cone in point R_2. A straight line segment that is drawn through point R_2 parallel to axis OX is a projection onto plane Π_2 of an auxiliary circle of radius S_1R_1. Point R_2 is a middle of this segment.

One can find the projections of other points (M, N, Q) onto plane Π_2, having drawn several generating lines of a cone. Fig.97 shows that points M, N, Q that belong to a hyperbola can be found with the help of the generating lines SA, SB and SC of a cone. Having connected points K_2, M_2, N_2, R_2, Q_2, C_2, we shall get a front projection of a hyperbola.

Problem 2. Construct a projection of a surface section of a right circular cone by front projecting plane α (fig.98).

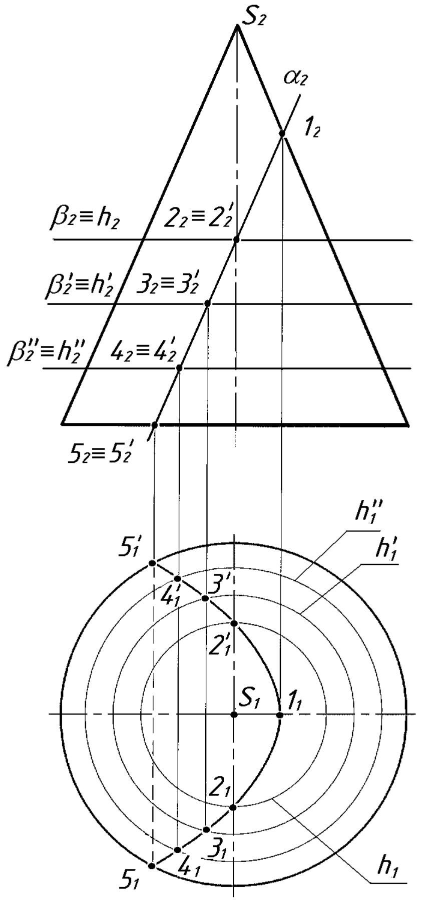


Fig. 98

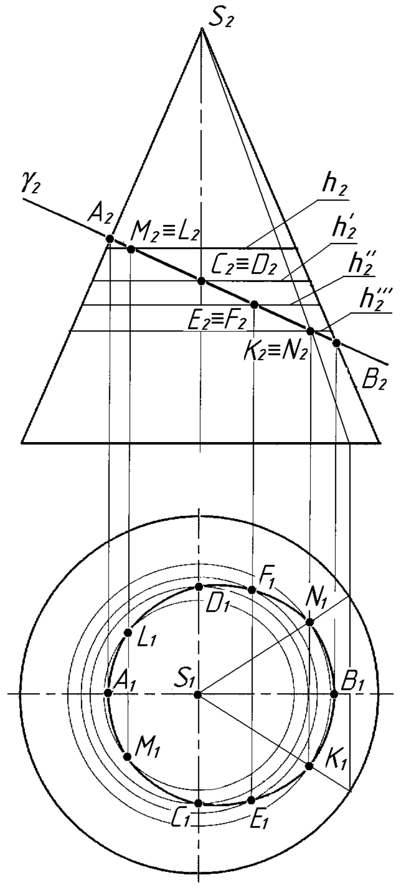


Fig. 99

Solving. As plane α is parallel to one of the extreme generating lines of a cone, we shall get a parabola in a section. A front projection of a parabola coincides with a trace – projection α_2 of an intersecting plane.

To construct a horizontal projection of a parabola, we draw several auxiliary horizontal planes (β, β', β''), each of them intersects a cone surface in circle and plane α – in a straight line that is perpendicular to Π_2. At the intersection of horizontal projections of these straight lines and horizontal projections of the corresponding circles we shall get points 2_1, $2'_1$, 3_1, $3'_1$, and 4_1, $4'_1$. We get horizontal projection 1_1 of a parabola top and also points 5_1, $5'_1$ that lie both on a parabola and on a circle of a cone base, having drawn a link line from points 1_2 and 5_2. If we connect points $5_1 – 1_1 – 5'_1$ with a smooth curve, we get a horizontal projection of a parabola. Hatching line 5_1 $5'_1$ is a horizontal projection of a straight line, on which plane α intersects a plane of a cone base.

Problem 3. Construct projections of a surface section of a right circular cone by front projecting plane α (fig.99).

Solving. As plane α is not perpendicular to a cone axis, we get an ellipse in a section, a big axis AB of this ellipse is projected onto plane Π_2 without deformation (A_2B_2), and a small axis CD of the ellipse is projected onto plane Π_2 into a point C_2D_2, located in the middle of a segment (A_2B_2). A size of the small axis (CD) is determined with a condition that CD $\in$ α (fig.99).

Through C_2D_2 we draw a front projection of the parallel of surface *h*. To construct its horizontal projection from a horizontal projection of a focus of ellipse S, we draw a circle of radius 1_22_2 and mark its intersection points C_1 and D_1 with a perpendicular that is put down from point $C_2 \equiv D_2$.

We get arbitrary points MN and FE with the help of the parallels of the surface according to *h'* and *h''*. Having connected the obtained points in consecutive order, we shall get a horizontal projection of a section – an ellipse.

The problem can also be solved with the help of the generating lines.

For this purpose we first draw front projections of the generating lines and then – horizontal projections through the selected points ($C_2 \equiv D_2$; $E_2 \equiv F_2$; $N_2 \equiv M_2$) on a front trace of plane α and cone vertex S_2. On link lines we find horizontal projections of these points on horizontal projections of the generating lines.

A natural size of a section figure can be found by a projection plane replacement. We draw plane Π_4 parallel to plane α_2 and replace Π_1 by Π_4. From points that lie on a section we draw link lines athwart X_{24} and on them from axis X_{24} we put a distance from these points to Π_2. Then we connect the obtained points with a smooth curve.

Evolvent of a surface of a right circular cone presents a circular sector, a radius of which is equal to the length of *l*=1S of a generating conic surface and central angle $\boldsymbol{\varphi} = (360^*r)/l$, where *r* is a circle radius.

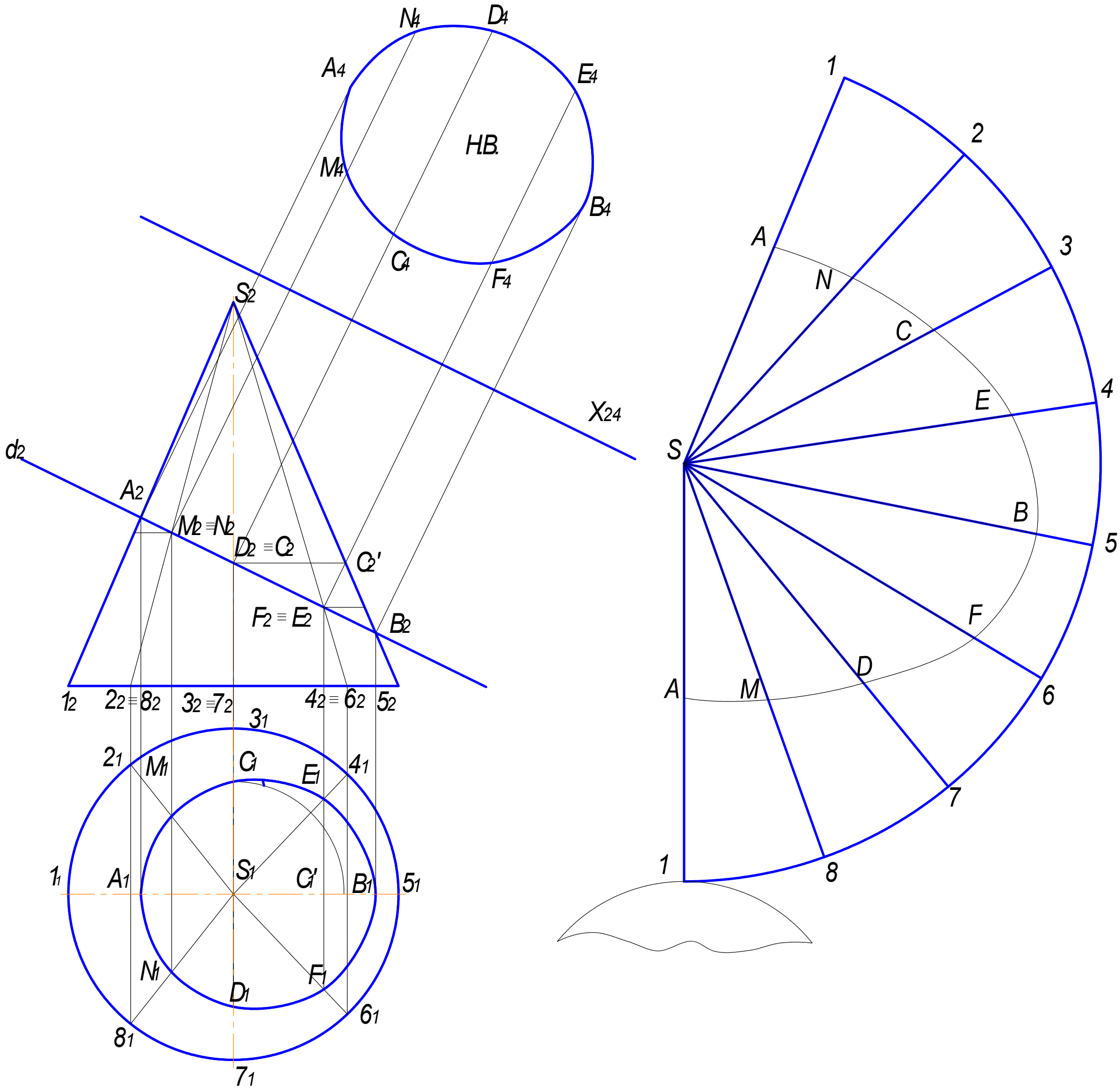


Fig.100

To draw a section line on a lateral cone surface, we put the generating lines (1 – 8) on the unfolded lateral cone surface, we determine and put the sizes of the segments of these generating lines. Points A and B lie on the natural sizes of the generating lines, that is why segments AS and BS are projected onto Π_2 also into their natural sizes and we put them without any change on the evolvent.

The length of the segments on other generating lines is determined by their rotation to the position, that is parallel to Π_2 (this construction is shown on generating line 3S for point C).

We connect the obtained points on the generating lines with a smooth curve. For the full evolvent of a cone we attach its base.

Problem 4. Construct a section of a right circular cylinder by general position plane α (fig.101a).

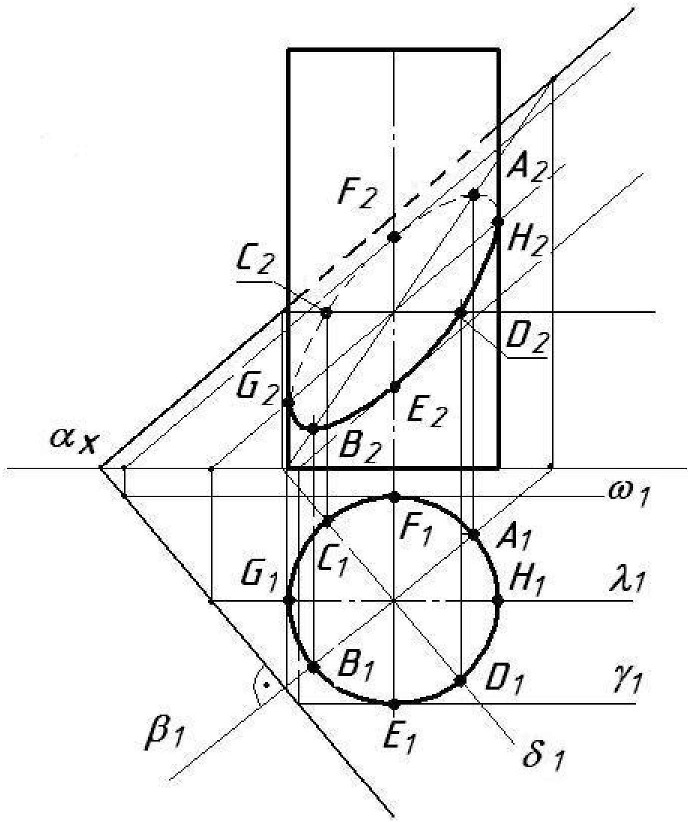


Fig.101a

Solving. As plane α is neither parallel to a cylinder axis, nor perpendicular to it, it will intersect a cylinder surface in an ellipse.

We draw front plane β through a cylinder axis. This plane intersects a cylinder in its extreme generating lines and plane α in its front line. The intersection of a front line and the extreme generating lines will determine two anchor points A and B that lie on the visibility limit of an ellipse.

We can find the anchor points – the closest one C and the furthest one D with the help of two front planes γ and δ that go through the generating lines of a cylinder. These planes intersect plane α also in a front line and the intersection of these front lines and the generating lines is given by the front projections of point C and D.

The highest point E and the lowest point F of an ellipse of section can be found with the help of horizontal projecting plane λ that is drawn through a cylinder axis athwart trace K_1.

Planes λ and α intersect in line 1 – 2 of the largest inclination of plane α in relation to Π_1. Plane λ intersects a cylinder surface in generating lines 3 – 4 and 5 – 6. The intersection of straight line 1 – 2 and straight lines 3 – 4 and 5 – 6 gives anchor points E and F.

Besides anchor points A, B, C, D, E, F we shall find points K and L. These points are constructed with the help of horizontal plane φ that intersects a cylinder surface in circle and plane α – in horizontal line 7 – 8. If we find K_1 and L_1 at the intersection of a horizontal projection of horizontal line 7_1 8_1 and a circle, we can find projections K_2 and L_2 on a trace of plane φ through the link lines.

We connect the obtained projections of the points on Π_2 with a smooth curve, taking into account the visibility.

The evolvent of a cylinder appears to be a right-angled rectangle, the length of which is equal to the length of a circle of a cylinder base (l=2πR) and the height is equal to the height of a cylinder (fig.101b).

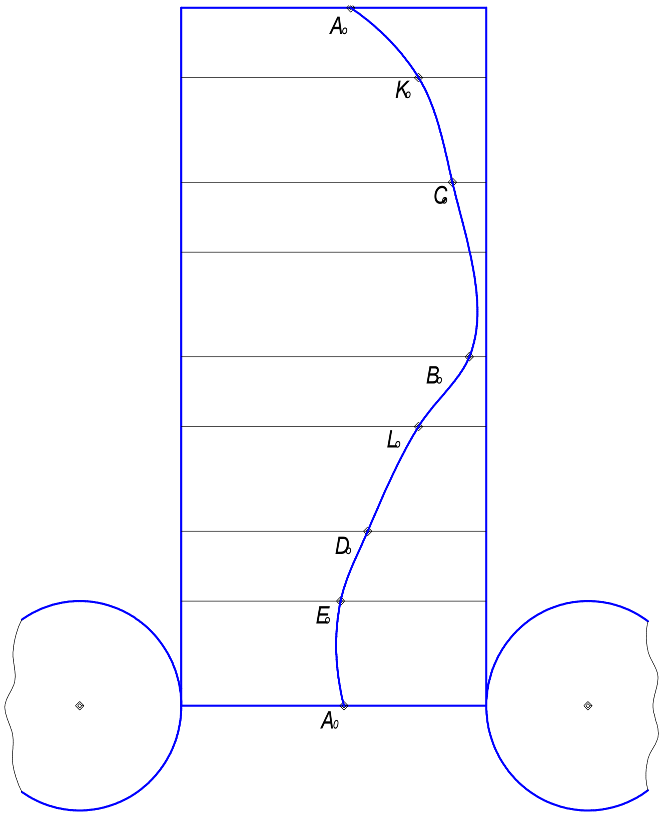


Рис. 101б.

Fig.101b

To put a section line on the evolvent, we should draw on it those generating lines of a cylinder, on which the section points lie.

Their height must be taken from plane Π_2. The obtained points are to be connected with a smooth curve. The upper and the lower cylinder bases should be attached to the evolvent of the lateral surface.

Problem 5. Construct a section of a four-angled prism by a general position plane (fig.102a).

Solving. We solve the problem by a projection plane replacement method. We draw new plane Π_4 athwart a horizontal projection of horizontal line (h_1) of plane (h×f).

We take two points P and F on a plane and move their coordinates from Π_2 to Π_4. Point F is chosen at random. Projecting plane (f_4×h_4) has been got. We also move a prism onto Π_4. To do it, we draw link lines athwart X_{14} from each point of a prism base ($A_1B_1C_1D_1$) from Π_1 to Π_4. A prism base lies on Π_1, that is why all the points of its base will lie on axis X_{14}. The prism height is transferred from Π_2 to Π_4.

A distance of the intersection of each edge and an intersecting plane is moved from Π_4 to Π_2, for example, $B_4B_4'=B_2B_2'$. The obtained front projections of the intersection points of each edge and a plane are connected by straight lines, taking into account the visibility.

We determine a natural size of a section by a planar movement method. For this purpose we place a plane of section which is projected onto Π_4 in a straight line (B_4', A_4', D_4', C_4') parallel to axis X_{14}. From each point of section we draw straight link lines athwart X_{14}. We shall get a natural size of a section figure at the intersection of these lines and the link lines that are drawn from the horizontal projections of the section points (A_1, B_1, C_1, D_1) parallel to X_{14}.

The evolvent of the prism presents a right-angled rectangle, the length of which is equal to the sum of the prism base sides (AB+BC+CD+AD). Each side on Π_1 is projected into its natural size, because the prism base lies on Π_1.

The rectangle height is equal to the prism height. Each prism edge is a horizontal projecting straight line which is projected onto Π_2 into its natural size. We attach the upper and the lower bases to the lateral surface. We put a section line (A_0 B_0 C_0 D_0 A_0) onto the evolvent (distance $A_0A^1=A_2A^1$).

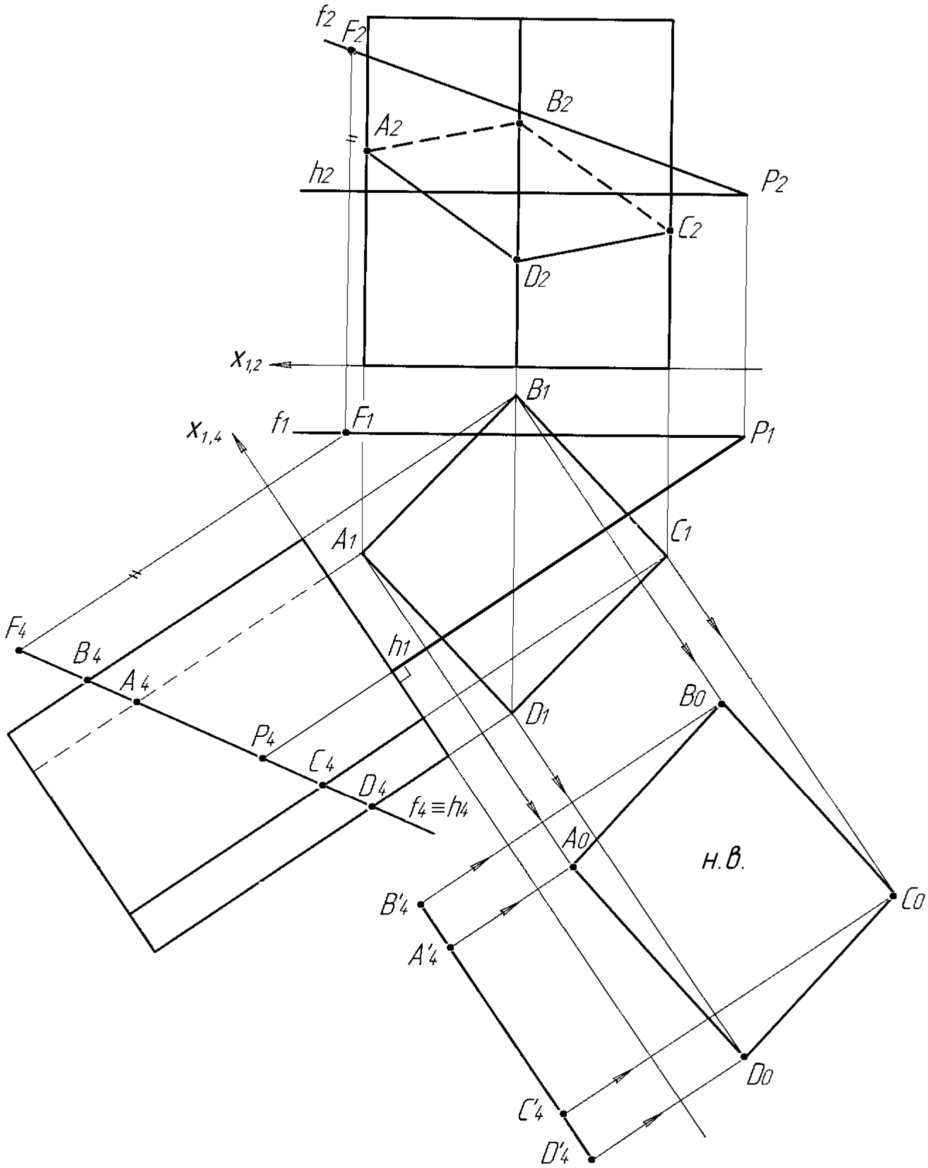


Fig.102a

Problem 6. Construct a section of a triangular pyramid by a general position plane (fig.103a).

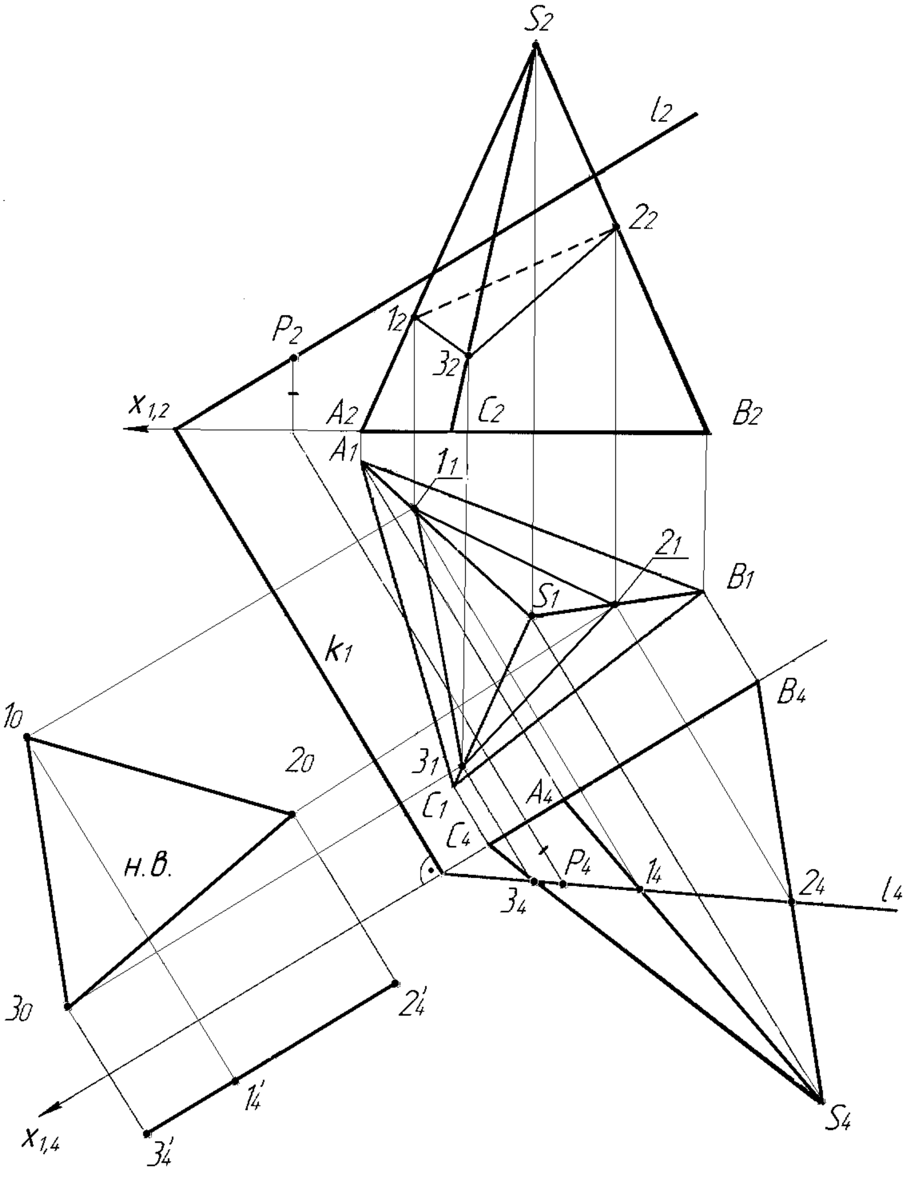


Fig.103a

Solving. We solve the problem by a projection plane replacement method in the following succession:

1. We turn a plane that is specified by traces into a projecting one on Π_4. For this purpose we draw an auxiliary plane Π_4 athwart horizontal trace b_1. On front trace l_2 we take an arbitrary point P and we move its coordinate along axis Z onto Π_4. Having connected a point of intersection of horizontal trace b_1 and axis X_{14} with point P_4, we shall get a projecting plane on Π_4.

2. We construct a pyramid on Π_4. From each point of a base and from a vertex of a pyramid we draw link lines onto Π_1 athwart X_{14}. Pyramid base ABC will be located on axis X_{14} and vertex S will lie on a distance that is equal to a distance from point S to Π_1.

3. We project the obtained section points 1_4, 2_4, 3_4 onto the corresponding edges through the link lines first onto Π_1 and then onto Π_2. Having connected the corresponding projections of points 1, 2, 3 by straight lines, we shall get a horizontal and a front projection of section. All the lines of section will be visible on Π_1. As edge ABC is invisible on Π_2, then line $1_2 - 2_2$ of section will also be invisible.

4. We construct a natural view of section by a planar movement method. For this purpose we move the section that is projected onto Π_4 into a straight line ($1_4 2_4 3_4$) onto a free place parallel to axis X_{14} without change of a distance between the points. At the intersection of the link lines from points 1_4, 2_4, 3_4 that are perpendicular to axis X_{14} and the link lines from points 1, 2, 3 that are parallel to axis X_{14} we shall get triangle $1_0 2_0 3_0$, i.e. a natural size of section.

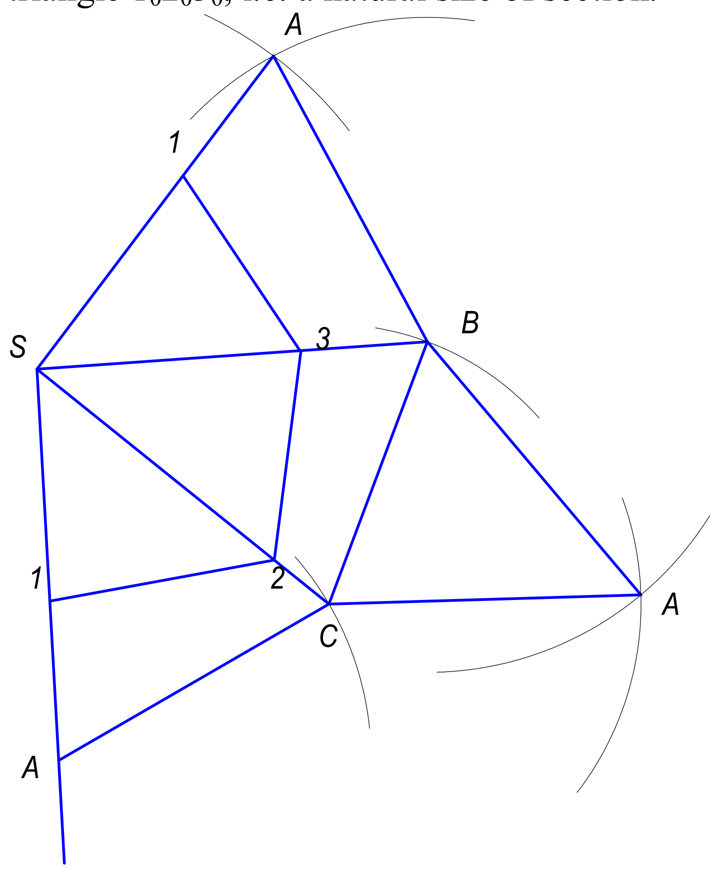


Рис. 103б.

Fig.103b

5. To make the evolvent of a pyramid lateral surface (fig.103b) with sides of triangular section that are drawn on its edges, we should find a natural size of each lateral edge and a segment on it. On fig.103 it is made with the help of a method of rotation around the axis, perpendicular to Π_1 that goes through point S. A pyramid base lies on Π_1, that is why sides AB, BC, AC have their natural size on Π_1. First, we construct the evolvent of edge $A_0S_0B_0$ according to three sides: $A_0B_0=A_1B_1$ and the lateral sides are equal to their natural sizes $A_0S_0=A_2S_2$, $B_0S_0=B_2S_2$. We construct triangle $A_0S_0B_0$ with the help of the compasses by putting marks. Then

we attach another triangle to side BS, taking into account that two other sides have the following sizes: side BC is equal to horizontal projection B_1C_1, side SC is equal to segment S_2C_2. We construct the third triangle the same way. As a result, we get an unfolded lateral surface of a pyramid. To find point 1 of a section line on the evolvent, it is necessary to transfer a front projection of point 1_2 on a front projection (fig.103a) parallel to axis X_{12} into a natural size of edge S_2A_2, and then to put a distance from S_2 to the moved point 1_2 on the evolvent (fig.103b) on edge SA – distance S. Similarly we find points 2 and 3 on the evolvent.

***Problem** 7.* 1. Construct a triangular prism section by a general position plane (m×n) (fig.104a). 2. Make the evolvent of a prism by putting a line of section. 3. Find a natural size of a section.

Solving. 1. We find intersection points of prism edges A, B, C and plane (m×n). Having connected intersection points A^1, B^1, C^1, we shall get the projections of a section, taking into consideration the edges visibility.

2. In the given problem on fig.104b the prism evolvent has been made by a method of a normal section.

To get the prism edges in their natural size, we draw an additional projection plane Π_4 athwart Π_1 and parallel to the prism edges. Having replaced Π_2 by Π_4, we get a prism on Π_4, the edges of which have a natural size. We also project a section figure A_4, B_4, C_4 on them.

3. To get a normal section, we draw plane α athwart the prism edges. In system $\Pi_4 \perp \Pi_1$ plane α is perpendicular to plane Π_4 and that is why a projection of a section figure on plane Π_4 lies on trace α_4. The obtained intersection points K, M, N of plane α and the edges on Π_4 are projected onto the corresponding edges on Π_1.

4. With the help of a planar movement method we find a natural size of a normal section – triangle $M_0N_0K_0$.

5. On a free format field we draw a straight line and successively put segments M_0N_0, N_0K_0, K_0M_0 on it. From points M_0, N_0, K_0 we draw perpendiculars to straight line $M_0 - M_0$ (fig.104b). We put segments $M_0A_0=M_4A_4$; $M_0Q_0=M_4Q_4$; $N_0C_0=N_4C_4$; $N_0F_0=N_4F_4$ and others on them. Then we draw broken straight lines $A_0C_0B_0A_0$ and $Q_0F_0P_0Q_0$. We put a section line on the evolvent. Segments $A_0A_0=A_4A_4$, $C_0C_0=C_4C_4$, etc. We connect points A_0, C_0, B_0 with a broken line. To the evolvent of a lateral surface of the prism we attach an upper and a lower bases that are projected onto Π_1 into their natural size.

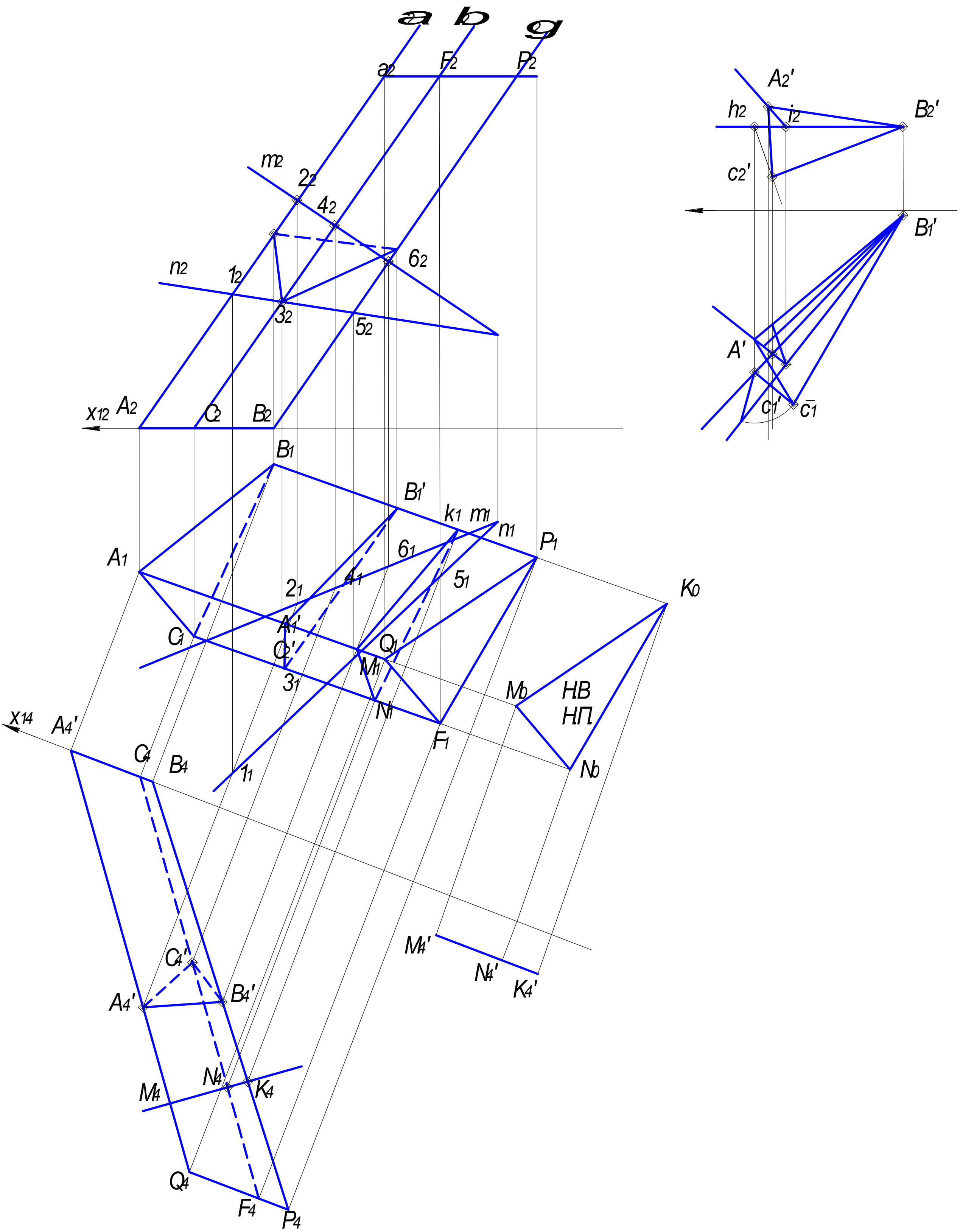


Рис.104 а

Fig.104a

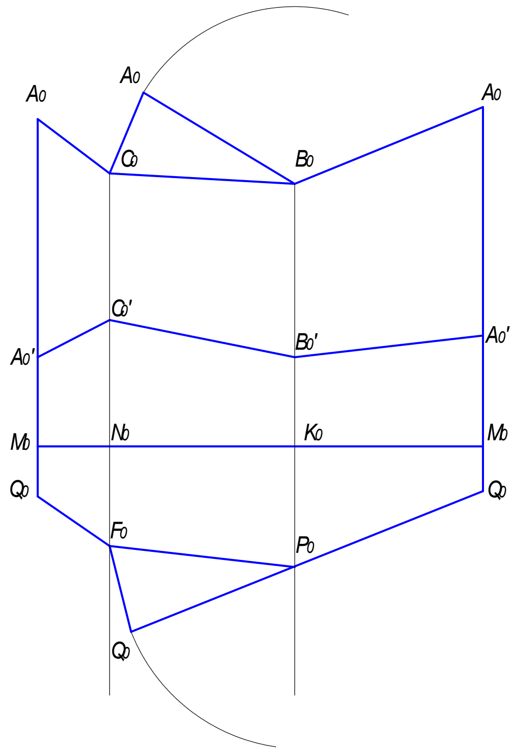


Рис. 104 б.

Fig.104b

6. We find a natural size of section A' B' C' by a method of rotation around the axis, parallel to plane Π_1.

The evolvent of a prism in the problem on fig.104b can be done with the help of unfolding (fig.105).

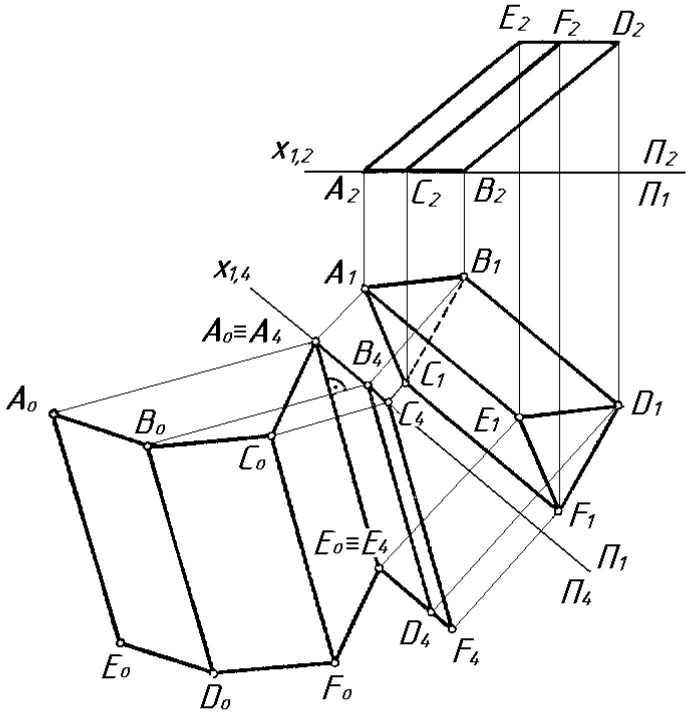


Fig.105

Having constructed the prism projection on plane Π_4, which is parallel to the prism edges, we draw from points A_4, B_4, C_4 the straight lines that are perpendicular to A_4Q_4, from point A_4 we draw a radius arc that is equal to A_1C_1 and we shall get point C_0 at the intersection with a straight line, which is drawn from C_4. From point C_0 we draw a radius arc which is equal to C_1B_1, and at the intersection with a straight line that is drawn from point B_4 we shall get point B_0, etc. ($A_0B_0=A_1B_1$). From points C_0, B_0, A_0 we draw straight lines, parallel to AO to the intersection with the corresponding straight lines, that are drawn from points F_4, P_4, Q_4.

On the evolvent we attach the prism bases and put the section line.

Questions to unit "Intersection of the edged and curved surfaces and a plane"

1. What is called a section?
2. What is the succession of a section constructing of the edged body by a projecting plane?
3. What is the succession of a section constructing of a surface of rotation by a projecting plane?
4. Name five conic sections.
5. Which methods are used to construct the surface sections by the general position planes?
6. What is called the evolvent of a surface?
7. Which methods are used to construct the evolvent of the surfaces?

Unit 13. INTERSECTION OF A STRAIGHT LINE AND A SURFACE

To construct the projections of the intersection points of a straight line and a surface, it is necessary:

1) to draw an auxiliary plane through the specified straight line;

2) to construct a line of intersection of the auxiliary plane and the specified surface;

3) to mark the intersection points of a straight line and a surface;

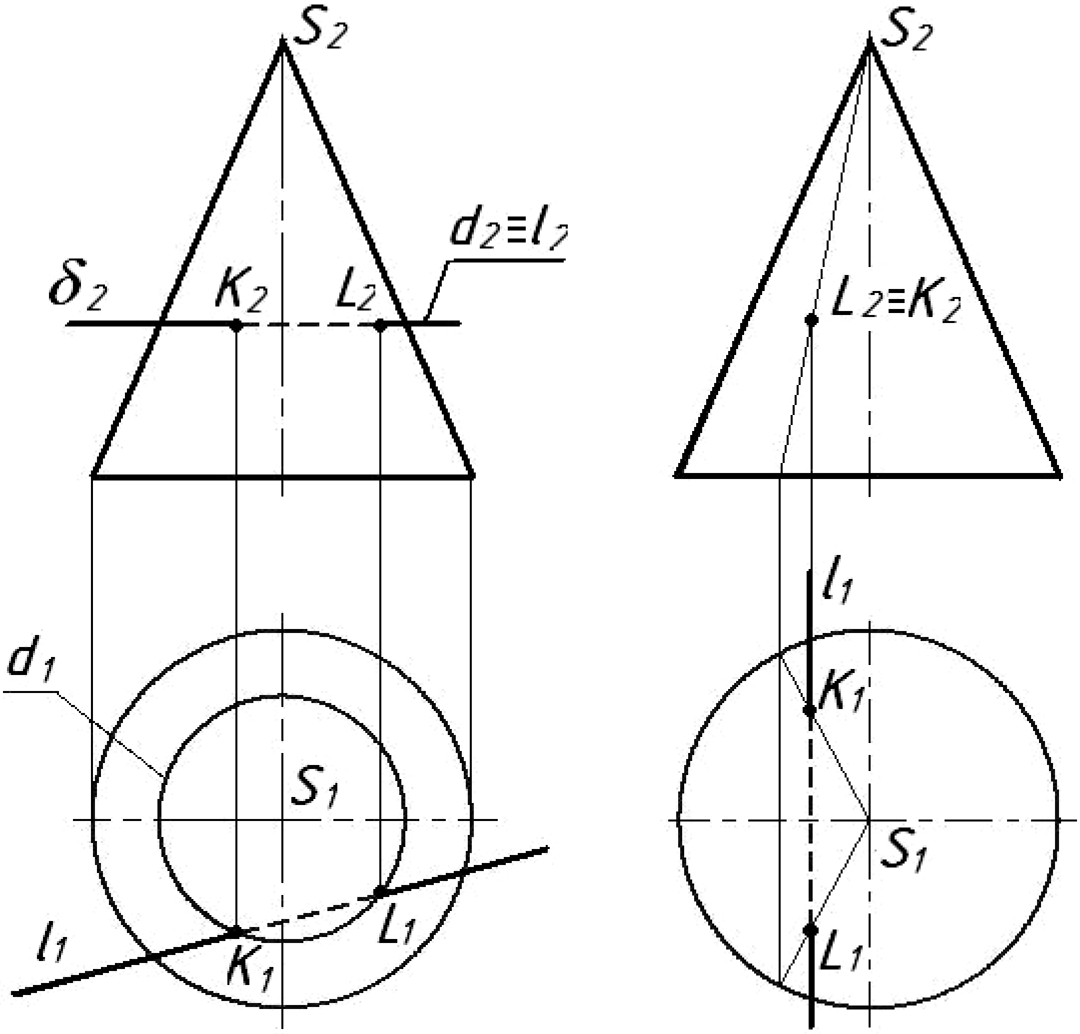
4) to determine the visibility of a straight line in relation to a surface.

While choosing the auxiliary plane we should take into consideration that this plane at the intersection with the surface should give such a line as a circle, a triangle, a parallelogram, etc.

Problem 1. Construct the intersection points of straight line *l* and a cone (fig.106).

Solving. Through straight line *l* (fig.106a) we draw a horizontal plane that makes up a circle on a cone surface by the section of a cone. We mark K_1 and L_1 where a horizontal projection of a circle intersects l_2.We construct K_2 and L_2 in a projection link. We determine the visibility of straight line *l*.

We draw a front projecting plane through straight line *l* (fig.106b). The front projecting plane goes through a vertex of a cone and makes up a triangle in a section on a cone surface.



а)   б)

Fig.106

Problem 2. Construct a line of the intersection of straight line *l* and a sphere. (fig.107).

Solving. Through straight line *l* (fig.107) we draw a front plane, which intersects a sphere and makes up a circle on a sphere surface.

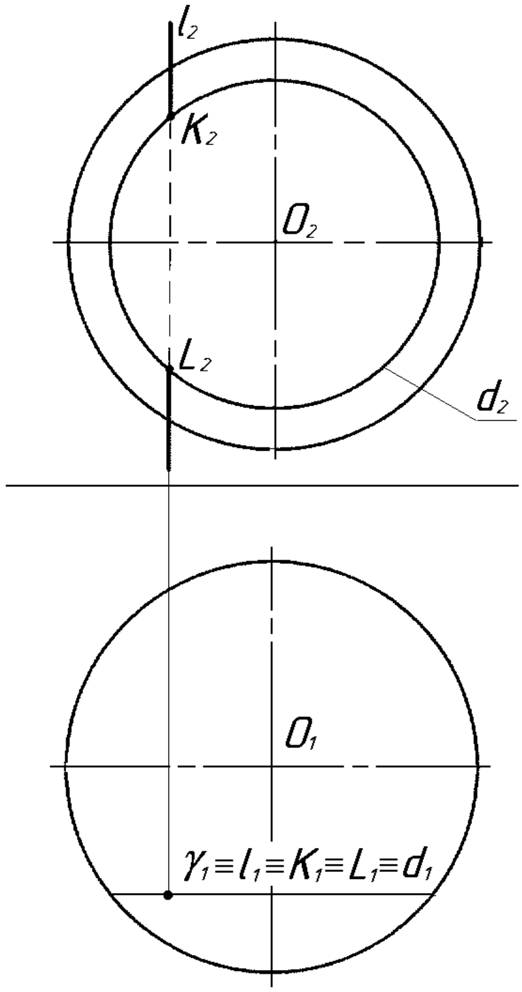


Fig.107, 108

Problem 3. Construct a line of the intersection of straight line *l* and a torus (fig.108).

Solving. Through straight line *l* we draw horizontal projecting plane α, we construct a line of the intersection of plane α and a torus and find the intersection points of straight line *l* and a torus. Then we determine the visibility.

Problem 4. Construct the points of the intersection of straight line *l* and a cone (fig.109).

To solve this problem, we may draw through *l* an auxiliary plane of a special position, which will make up a curve by the section of a cone. But the simplest way to solve this problem is to draw an auxiliary plane of a general position through straight line *l*. This plane must by all means go through a vertex of a cone, making up a triangle on a cone surface by its section.

Solving. 1. Through vertex S of a cone we draw straight line *m* that intersects straight line *l* in point A. We got a plane, specified by two straight lines that intersect α (*m*×*l*).

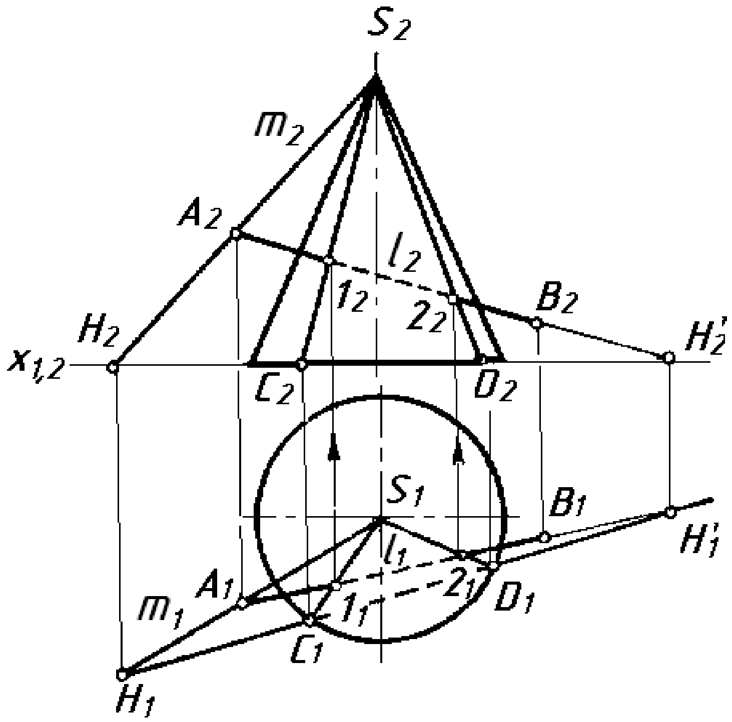


Fig.109

2. We construct a horizontal trace of an intersecting plane. For this purpose, by two intersecting straight lines *l* and *m* we determine horizontal traces of straight lines *l* and *m* and connect them (line M_1N_1).

3. Taking into account that a base of a cone and a horizontal trace of an intersecting plane lie in Π_1, we mark the points of the intersection of an intersecting plane trace and a cone base. Having connected these points with the vertex of a cone, we shall get a section of a cone by an auxiliary plane – a triangle.

4. We mark the points of the intersection of straight line *l* and a section and determine the visibility.

Problem 5. Construct the points of the intersection of straight line *l* of a general position and a cylinder (fig.110).

Solving. In this problem we take a general position plane as an auxiliary plane that is parallel to the generating lines of a cylinder. This plane is specified by two straight lines $l \times m$. A parallelogram is made up on the cylinder surface due to the section of a cylinder by such a plane. We mark the points of the intersection of straight line *l* and the cylinder and then we determine the visibility.

Questions to unit "Intersection of a straight line and a surface"

1. What is the succession of finding the points of the intersection of a straight line and a surface?

2. Which planes are advisable to use to construct the points of the intersection of a straight line and a surface?

3. What is the succession of constructing the points of the intersection of a straight line of a general position and a cone?

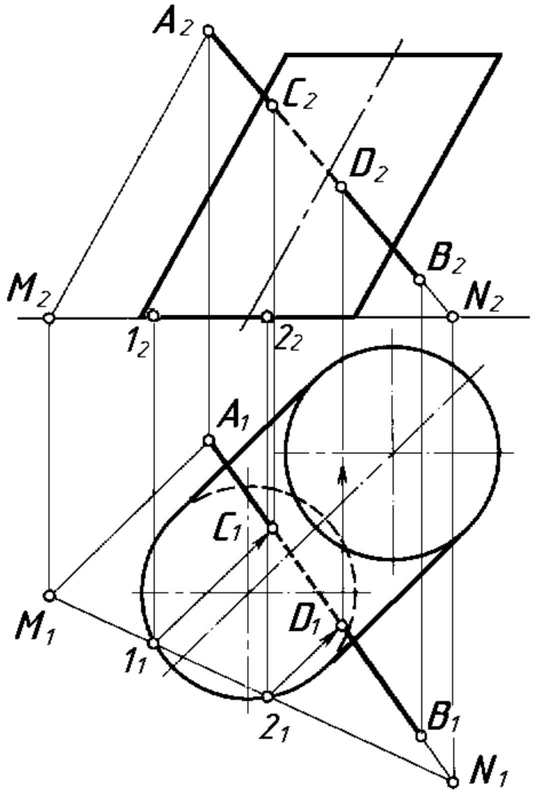


Fig.110

Unit 14. INTERSECTION OF SURFACES

A line of the mutual intersection of two surfaces is the line which simultaneously belongs to both surfaces that intersect. To construct such a line, it is necessary to find several points that simultaneously belong to both surfaces. Herewith, they use the auxiliary intersecting surfaces. Planes or spheres often serve as the auxiliary surfaces. Depending on this there are two methods of constructing the intersection lines of the surfaces – the intersecting planes and spheres.

14.1. A method of auxiliary intersecting planes

To construct an intersection line of two surfaces with the help of this method, they use the auxiliary intersecting planes of a special position. Let us study this method to solve a problem to construct a line of the intersection of a cylinder and a hemisphere (fig.111).

We start solving the problem with the analysis of its conditions. As a cylinder has a front projecting position, a line of the intersection is projected onto Π_2 onto a circle – a cylinder projection. The highest points are 1 and 2. To construct the horizontal projections of points 3 and 4, we shall use horizontal intersecting plane α. A section of this plane and a sphere makes a circle with radius R_1. A section of this plane and a cylinder makes a straight line. The place where this straight line and circle R_1 intersect, is marked by points 3 and 4 that are common for the sphere, the intersecting plane and the cylinder, i.e. these points belong to the line of the intersection of the sphere and the cylinder. Points 5 – 8 lie on the visibility limit for the cylinder. For their construction we use intersecting plane β that in a section with the sphere makes a circle with radius R_2 and with the cylinder – a rectangle. Similarly, with the help of intersecting plane γ we find projections of intermediate points 9 – 12.

The obtained points are connected with a smooth curve, taking into account their visibility. The intersecting planes method can also be used while constructing the intersection lines of a surface of rotation and the sided surfaces.

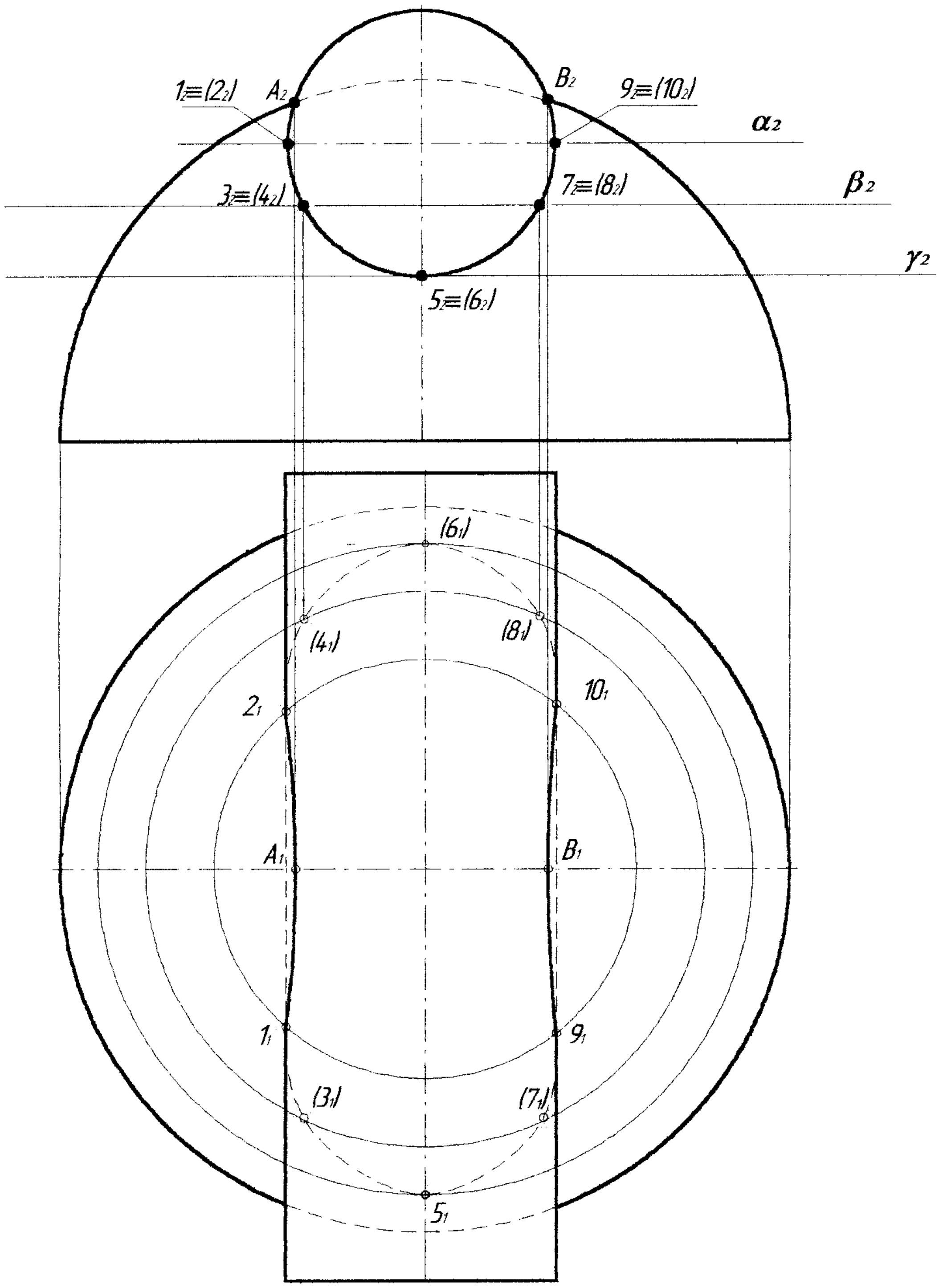


Fig.111

14.2. A method of spheres

Before we examine the construction of the line of two surfaces with the help of spheres, we should first study the coaxial surfaces (fig.112).

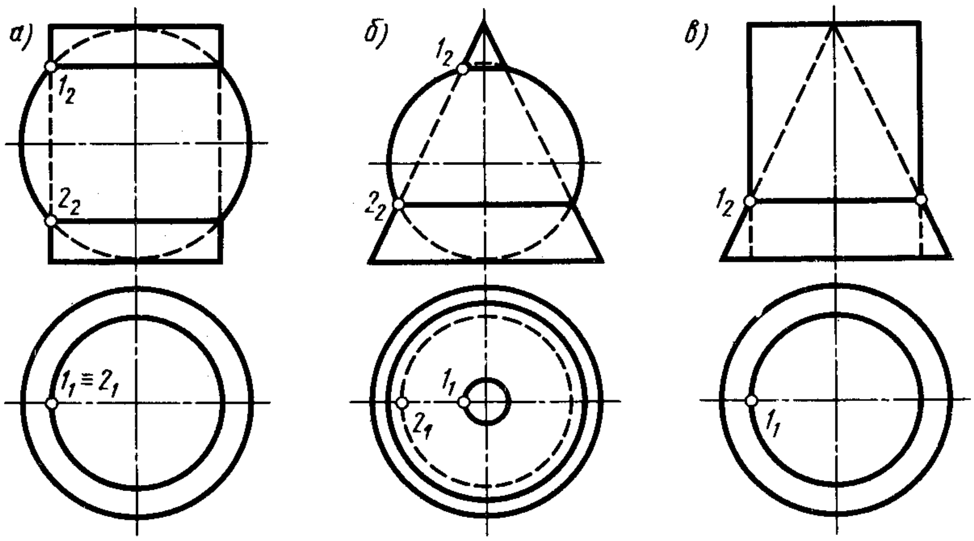


Fig.112

The surfaces of rotation are called the coaxial surfaces, if they have a common axis of rotation. If a sphere centre lies on an axis of rotation of any surface, such a pair of surfaces is also called coaxial. Two coaxial surfaces always intersect in a circle (fig.112-114). If a sphere intersects any surface of rotation and a sphere centre lies on an axis of rotation of this surface, the line of the intersection of these surfaces is a circle.

A section can make as many circles, as many outlines of spheres intersect an outline of a surface of rotation (fig.112-114). If an axis of a surface is parallel to a projection plane or perpendicular to it, these circles are projected onto a projection plane as the straight lines.

14.3. A method of concentric spheres

To use a method of concentric spheres, one should fulfill the following conditions:

1. Both intersecting surfaces should be the surfaces of rotation;
2. The axes of the surfaces of rotation should intersect (lie in one plane);
3. A plane, in which the axes of rotation intersect, should be parallel to any projection plane.

Problem. Construct the intersection line of a cylinder and a straight circular cone (fig.113).

Solving. We determine the points of the intersection of the outlines of the specified surfaces. We draw an auxiliary sphere of radius R_{min} that fits into one of the surfaces and intersects another one. In the given problem a sphere of radius R_{min} fits into a cone. Sphere R_{min} has a common tangent circle with the cone. Sphere R_{min} intersects the cylinder in two circles with diameter CD and MN.

At the intersection of circle AB and circles CD and MN we mark points 5, 5^1 and 6, 6^1. To construct the auxiliary points, we take a sphere, the radius of which is a little bigger than R_{min}, and we intersect the cone and the cylinder by this sphere.

By the intersection of the sphere and the cone a circle of diameter EF will be made up, and by the intersection of the sphere and the cylinder two circles with diameters GK and QL will appear. At the intersection of these circles we mark points 7, 7^1 and 8, 8^1, which we connect with a smooth curve. We construct a projection of the line of the intersection onto Π_1.

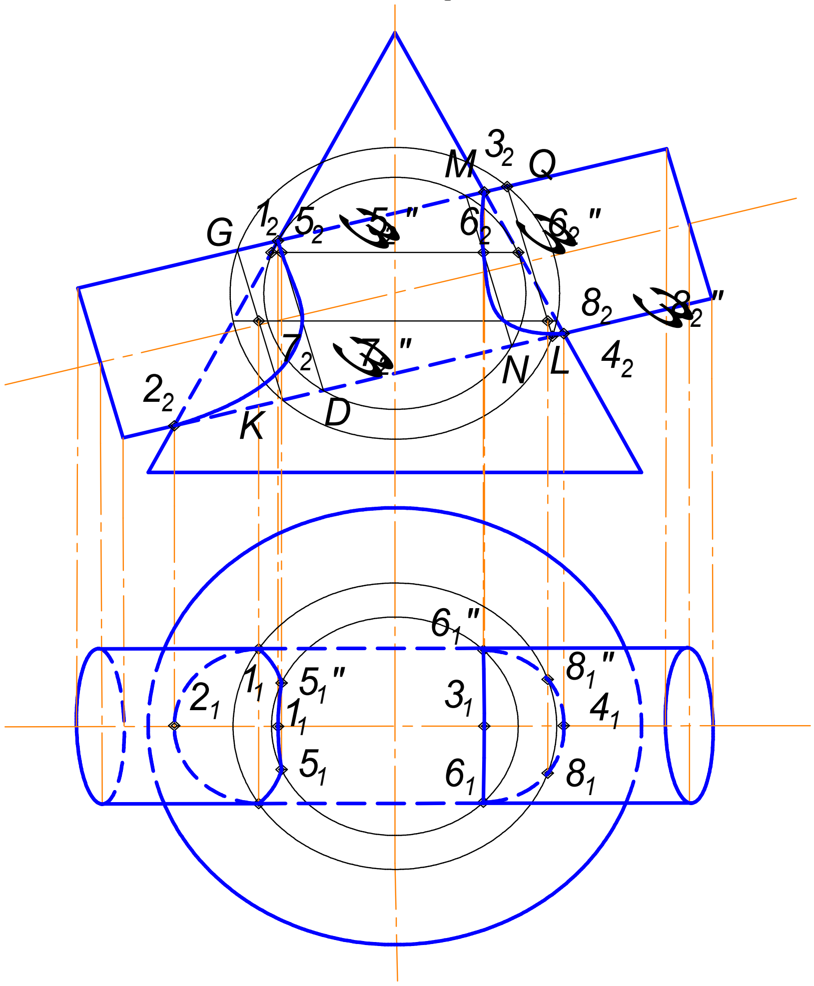


Рис.115

Fig.113

14.4. A method of eccentric spheres

To solve the problems to intersect the surfaces by this method, one should change the positions of the centres of the auxiliary spheres: they have to lie on an axis of a surface of rotation.

Problem. Construct a line of the intersection of a cylinder and a sphere (fig.114).

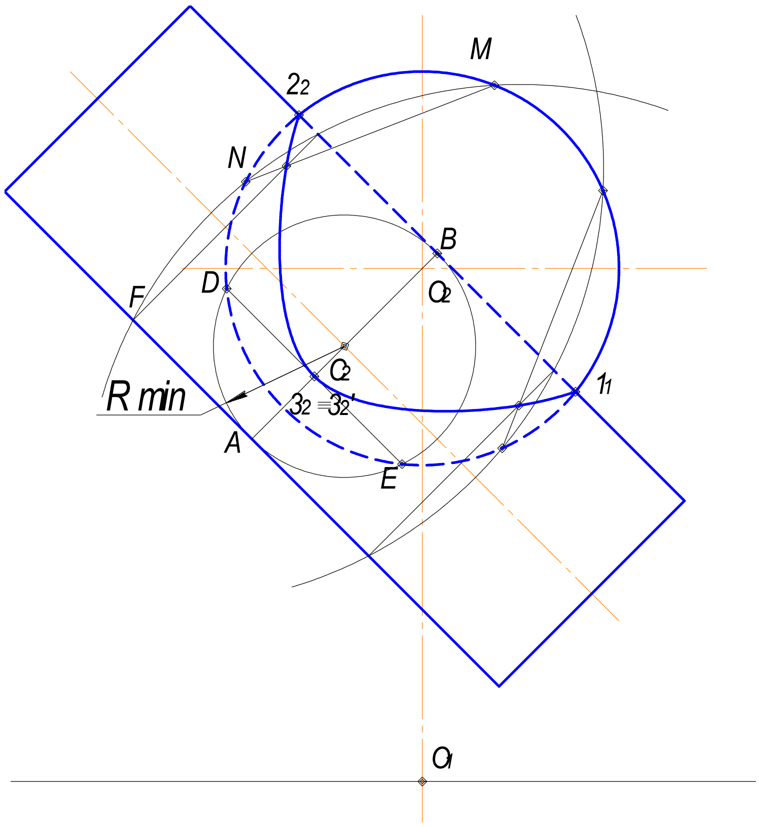


Рис. 116

Fig.114

Solving. 1. We mark the points at the intersection of the outlines of the surfaces. We draw an auxiliary sphere R_{min}, which fits into the cylinder. This sphere and the cylinder have a common tangent circle of diameter AB, and it intersects the sphere in a circle of diameter DE. At the intersection of these circles we mark points 3 and 3^1.

2. To determine the other points of the line of the intersection, it is necessary to draw several auxiliary spheres more, the centres of which lie on the axis of a cylinder of rotation.

3. We connect the found points with a smooth curve and construct a projection of a line of the intersection onto Π_1.

Problem. Construct a line of the intersection of a cone and a torus (fig.115).

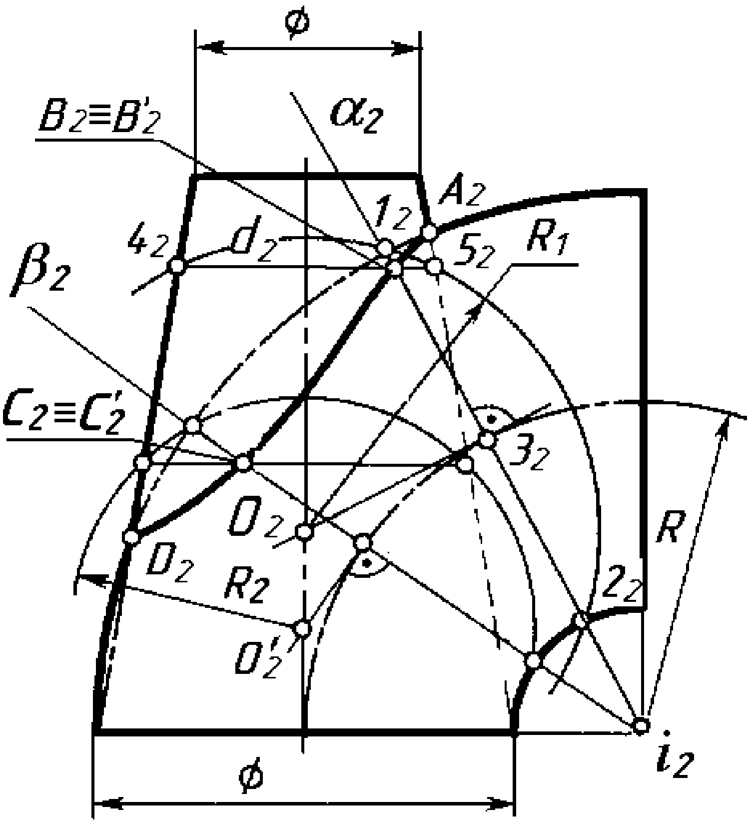


Fig.115

Solving. 1. We mark the points of the intersection of the outlines of the surfaces.

2. Through a rotation axis of a torus we draw an auxiliary plane α that cuts the torus in a circle of diameter AB. At the intersection of plane α and a circle of a torus axis we mark point P. From this point we draw a perpendicular to plane α to the intersection with a cone axis in point C. Then we draw a sphere of radius $R_{сф}$ from the centre in point C of radius CA. This sphere intersects the cone in a circle of diameter DE. Circles AB and DE belong to one sphere, therefore, they intersect in point $3 \equiv 3^1$.

To find one more point, it is necessary to use one more plane β and to repeat all the actions from the beginning. Then one should connect the found points with a smooth curve and construct a projection of a line of the intersection onto Π_1.

14.5. Monge's theorem

If in two intersecting surfaces of rotation one can fit a sphere, then a line of the intersection of these surfaces disintegrates into two curves – ellipses (fig.116).

Questions to unit "Intersection of surfaces"

1. Which methods are used to construct a line of the mutual intersection of surfaces?

2. Which method of constructing a line of the mutual intersection of surfaces is considered to be universal?

3. In what cases do they use a method of concentric spheres?

4. In what cases do they use a method of eccentric spheres?

5. Formulate Monge's theorem.

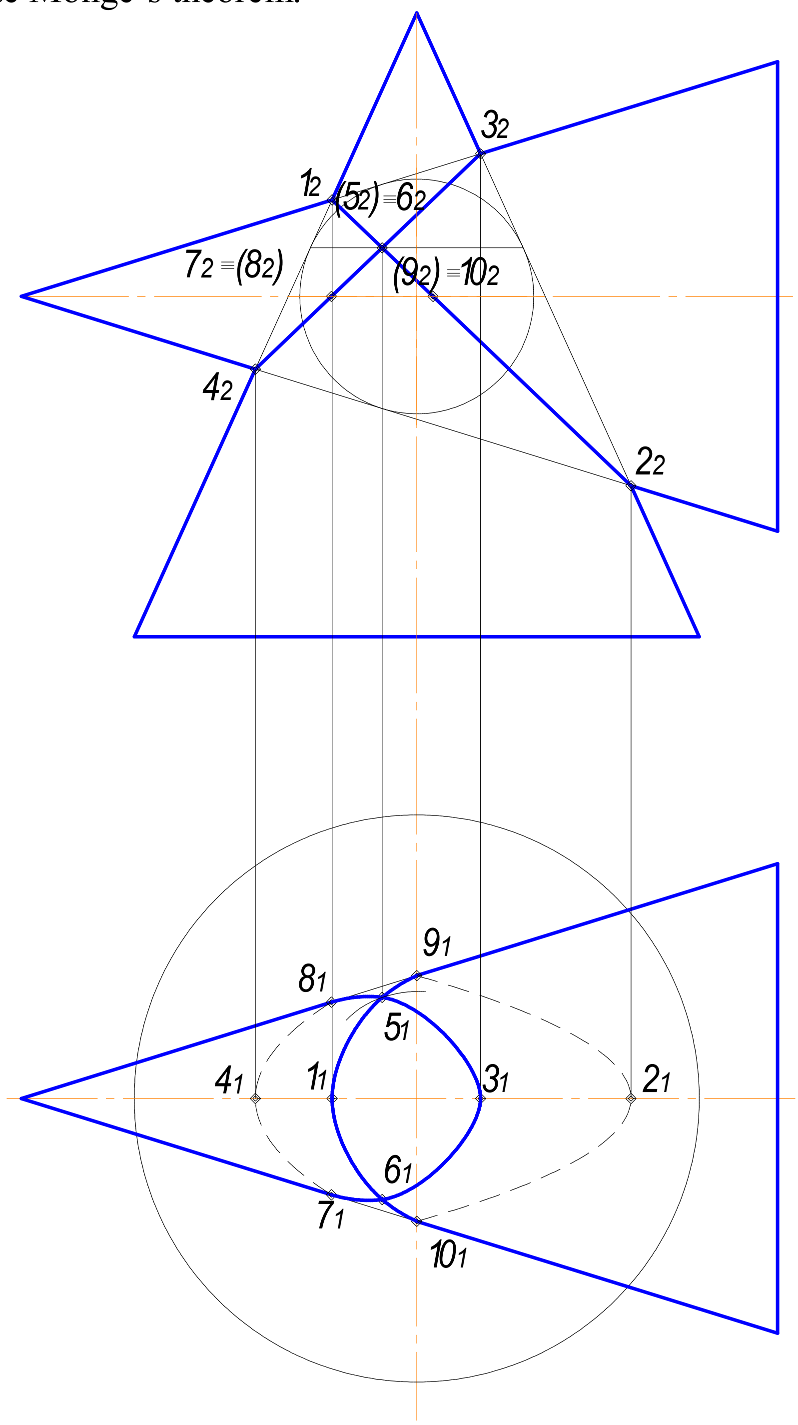


Рис. 118

Fig.116

GLOSSARY

A vertex of a polyhedron is a point at which the adjacent edges of a polyhedron converge.

A vertex of a polygon is a point at which two adjacent sides of a polygon converge.

An axis of rotation is a fixed straight line, around which a generating line of a surface rotates.

A projection axis is a line of the intersection of the projection planes in a rectangular system. At the intersection of a horizontal and a front projection planes an axis is formed. Axis OX is an axis of a front and a profile projection planes – axis O, axis OY is an axis of a horizontal and a profile projection planes.

A helix surface is formed by the motion of segment AB, both ends of which move on the cylindrical helixes, and an angle between the segment and an axis of a cylinder remains constant all the time.

A helix is a trajectory of the motion of a point, which uniformly glides forward along the generating line of any surface of rotation, if this generating line itself uniformly rotates around the axis of this surface. They distinguish helixes of the constant and variable motion. The most common helixes that are used in engineering are cylindrical helixes and conic helixes of the constant motion.

A conic helix is a trajectory of the motion of a point, which uniformly moves forward along the generating line of a circular cone, which uniformly rotates around an axis of a cone. A horizontal projection of a conic helix is Archimedes spiral, and a front projection is a damped sinusoidal curve with the wavelength that decreases. By unfolding of a conic surface a helix is also transformed into Archimedes spiral. A conic helix can be right-handed and left-handed.

A cylindrical helix is a trajectory of the motion of a point, which uniformly moves forward along the generating line of a straight circular cylinder, while the generating line uniformly rotates around an axis of a cylinder.

ENGLISH-UKRAINIAN VOCABULARY

A

abscissa – абсциса
accidental surface – незакономірна поверхня
adjacent – суміжний
adjacent edge – сусіднє ребро
adjacent side – сусідня сторона
algorithm – алгоритм
algorithmic part of a determinant (APD) – алгоритмічна частина визначника (АЧВ)
anchor point – опорна точка
angle – кут
angle of inclination – кут нахилу
applicant – аплікатa
arbitrary – довільний
arrow – стрілка
athwart – перпендикулярно
at random – довільно
attach – добудувати
auxiliary – допоміжний
availability – наявність

B

back – задній
base – основа
be determined – визначатись
be located, be placed – бути розташованим
be projected – проектуватись
belong – належати
broken straight line – ламана пряма лінія

C

cathetus – катет
channel surface – каналова поверхня
circle – коло
circular cone – круговий конус
circular cylinder – круговий циліндр

circular sector – круговий сектор
circular surface – циклічна поверхня
closed helicoid – закритий гелікоїд
coaxial – співосний
coincide – співпадати
common – спільний
common tangent circle – спільне дотичне коло
compasses – циркуль
competitive – конкуруючий
complex drawing – комплексний рисунок
concentric sphere – концентрична сфера
cone – конус
cone vertex – вершина конуса
conic helix – гвинтова лінія конічна
conic surface – конічна поверхня
conoid – коноїд
constant – незмінний
constant radius – постійний радіус
construct – побудувати
construction – побудова
continuous progressive movement – переміщення безперервне поступальне
contour – контурний
converge – сходитись
conversion – перетворення (креслення)
coordinate – координата
correspond to smth. – відповідати чому-небудь
corresponding – відповідний
crosslying – мимобіжний
curve – крива лінія
curved edge – ребро звороту
curved surface – крива поверхня
curvilinear – криволінійчастий
cylinder – циліндр
cylindrical helix – гвинтова лінія циліндрична
cylindrical surface – циліндрична поверхня
cylindroid – циліндроїд

D

damped sinusoidal curve – затухаюча синусоїдальна крива
deformation – спотворення
depicting – зображення
determine – визначати
diagram – епюр

diameter – діаметр
direction – напрямок
direction of movement – напрям переносу
drawing – креслення

E

eccentric sphere – ексцентрична сфера
edge – ребро
edged body – гранне тіло
edged surface – гранна поверхня
ellipse – еліпс
ellipsoid – еліпсоїд
equator – екватор
evolvent – розгортка
extension – продовження
extreme point – крайня точка

F

facet – грань
figure – фігура
fit into smth. – вписуватись
fixed – нерухомий
fixed straight line – нерухома пряма
fold – складка
folded surface – нерозгортна поверхня
form – форма
four-angled prism – чотирикутна призма
front – передній
front straight line – фронталь

G

generating line – твірна лінія
geometric image – геометричний образ
geometric part of a determinant (GPD) – геометрична частина визначника (ГЧВ)
guiding line – напрямна лінія

H

half-plane – напівплощина
hatching line – штрихова лінія
height – висота
helicoid – гелікоїд (гелісоїд)

helix – гвинтова лінія
helix movement – переміщення гвинтове
helix surface – гвинтова поверхня
hemisphere – півсфера
horizontal straight line – горизонталь
hyperbola – гіпербола
hyperbolic paraboloid – гіперболічний параболоїд
hyperboloid – гіперболоїд
hyperboloid of rotation of one sheet – однополосний гіперболоїд обертання
hyperboloid of rotation of two sheets – двополосний гіперболоїд обертання
hypotenuse – гіпотенуза

I

image – образ
improper centre – невласний центр
incidence – інцедентність
inclination – нахил
infinitely – безкінечно
initial position – первісне положення
in pairs – попарно
intersection – перетин
intersection line – лінія перетину
invisible – невидимий

J

K

kinematic – кінематичний

L

lateral – бічний
lateral edge – бічне ребро
lateral surface – бокова поверхня
length – довжина
level straight line – пряма рівня
lie – знаходитись, розміщуватись
line – лінія
line of the largest inclination – лінія найбільшого ухилу
line of the largest slope – лінія найбільшого скату
link line – лінія зв’язку
location – розміщення
logical surface – закономірна поверхня

lower – нижній

M

mark – позначати
mark – позначка
measure – заміряти, виміряти
meridian – меридіан
meridional – меридіональний
mesh of moving surface – сітка поверхні переносу
metric – метричний
move – переміщувати
move in a circle – переміщуватись по колу
movement – переміщення
moving surface – поверхня переносу

N

natural size – натуральна величина
neck – горловина

O

oblique angle – гострий кут
oblique-angled – косокутний
oblique helicoid – косий гелікоїд
oblique plane – коса площина
obtained point – отримана точка
open helicoid – відкритий гелікоїд
ordinate – ордината
origin – початок координат
outline – обрис

P

parabola – парабола
parabola top – вершина параболи
paraboloid – параболоїд
parallel – паралель
parallel – паралельний
parallel position – паралельність
parallelogram – паралелограм
perpendicular – перпендикуляр
perpendicular – перпендикулярний
perpendicularity – перпендикулярність
planar movement – плоскопаралельне переміщення

plane – площина
plane image – плоске зображення
plane of parallelism – площина паралелізму
point – точка
point by arrows – вказувати стрілками
polygon – многокутник
polyhedron – многогранник
positional – позиційний
positive value – позитивне значення
prism – призма
problem – задача
profile straight line – профільна пряма
projecting ray – проекціюючий промінь
projecting straight line – проекціююча пряма
projection – проекціюваня
projection axis – вісь проекцій
projection plane – площина проекцій
proper centre – власний центр
property – властивість
put down – опускати
pyramid – піраміда

Q

quadrangle – чотирикутник
quadrant – квадрант
quarter – чверть

R

radius – радіус
radius arc – дуга радіуса
ray – промінь
rectangle – прямокутник
rectilinear – прямолінійчастий
replace – замінити
replacement – заміна
right-angled – прямокутний
right-angled triangle – прямокутний трикутник
right circular cone – прямий круговий конус
right helicoid – прямий гелікоїд
ring helicoid – кільцевий гелікоїд
rotate – розвертати, обертати
rotate clockwise – повертати за годинниковою стрілкою
rotate counterclockwise – повертати проти годинникової стрілки

rotation – обертання
rotation axis – вісь обертання

S

section – переріз
section line – лінія перерізу
segment – відрізок
side facet – бокова грань
size – величина
smooth curve – плавна крива лінія
solving – розв'язання
space – простір
spatial – просторовий
specified – заданий
specified direction – заданий напрям
specify – задавати
sphere – сфера
spiral – спіраль
straight line – пряма
succession – послідовність
superposition – суміщення
surface – поверхня
surface determinant – визначник поверхні
surface of rotation – поверхня обертання

T

tangent – дотичний
theorem – теорема
torus – тор
trace – слід
trajectory of motion – траєкторія руху
transformation – перетворення (чого-небудь у що-небудь)
triangle – трикутник
triangular prism – трикутна призма
triangular pyramid – трикутна піраміда
tubular surface – трубчаста поверхня
two-facet – двогранний

U

unfolded surface – розгортна поверхня
unfolding – розкатка
uniformly – рівномірно

upper – верхній

V

variable radius – змінний радіус
visibility – видимість
visibility limit – границя видимості
visible – видимий

W

X

Y

Z

zero – нуль

UKRAINIAN-ENGLISH VOCABULARY

А

абсциса – abscissa
алгоритм – algorithm
алгоритмічна частина визначника (АЧВ) – algorithmic part of a determinant (APD)
апліката – applicant

Б

безкінечно – infinitely
бічне ребро – lateral edge
бічний – lateral
бокова грань – side facet
бокова поверхня – lateral surface
бути розташованим – be located, be placed

В

величина – size
верхній – upper
вершина конуса – cone vertex
вершина параболи – parabola top
взаємне положення – mutual position
видимий – visible

видимість – visibility
висота – height
визначати – determine
визначатись – be determined
визначник поверхні – surface determinant
відповідати чому-небудь – correspond to smth.
відповідний – corresponding
відрізок – segment
вісь – axis
вісь обертання – rotation axis
вісь проекцій – projection axis
вказувати стрілками – point by arrows
власний центр – proper centre
властивість – property
вписуватись – fit into smth.

Г

гвинтова лінія – helix
гвинтова лінія конічна – conic helix
гвинтова лінія циліндрична – cylindrical helix
гвинтова поверхня – helix surface
гелікоїд (гелісоїд) – helicoid
гелікоїд відкритий – open helicoid
гелікоїд закритий – closed helicoid
гелікоїд кільцевий – ring helicoid
гелікоїд косий – oblique helicoid
гелікоїд прямий – right helicoid
геометрична частина визначника (ГЧВ) – geometric part of a determinant (GPD)
геометричний образ – geometric image
гіпербола – hyperbola
гіперболічний параболоїд – hyperbolic paraboloid
гіперболоїд – hyperboloid
гіпотенуза – hypotenuse
горизонталь – horizontal straight line
горловина – neck
границя видимості – visibility limit
гранна поверхня – edged surface
гранне тіло – edged body
грань – facet

Д

двогранний – two-facet

двополосний гіперболоїд обертання – hyperboloid of rotation of two sheets
діаметр – diameter
добудувати – attach
довжина – length
довільний – arbitrary
довільно – at random
допоміжний – auxiliary
дотичний – tangent
дуга радіуса – radius arc

Е

екватор – equator
ексцентрична сфера – eccentric sphere
еліпс – ellipse
еліпсоїд – ellipsoid
епюр – diagram

Є

Ж

З

задавати – specify
заданий – specified
заданий напрям – specified direction
задача – problem
задній – back
закономірна поверхня – logical surface
заміна – replacement
замінити – replace
заміряти, виміряти – measure
затухаюча синусоїдальна крива – damped sinusoidal curve
змінний радіус – variable radius
знаходитись, розміщуватись – lie
зображення – depicting

И

І

інцедентність – incidence

Ї

Й

К

каналова поверхня – channel surface
катет – cathetus
квадрант – quadrant
кінематичний – kinematic
коло – circle
комплексний рисунок – complex graphic
конічна поверхня – conic surface
конкуруючий – competitive
коноїд – conoid
контурний – contour
конус – cone
концентрична сфера – concentric sphere
координата – coordinate
коса площина – oblique plane
косокутний – oblique-angled
крайня точка – extreme point
креслення – drawing
крива лінія – curve
крива поверхня – curved surface
криволінійчастий – curvilinear
круговий конус – circular cone
круговий сектор – circular sector
круговий циліндр – circular cylinder
кут – angle
кут гострий – oblique angle
кут нахилу – angle of inclination
кут прямий – right angle

Л

ламана пряма лінія – broken straight line
лінія – line
лінія зв’язку – link line
лінія найбільшого скату – line of the largest slope
лінія найбільшого ухилу – line of the largest inclination
лінія перерізу – section line
лінія перетину – intersection line

М

меридіан – meridian
меридіональний – meridional
метричний – metric
мимобіжний – crosslying
многогранник – polyhedron
многокутник – polygon

Н

належати – belong
напівплощина – half-plane
напрям переносу – direction of movement
напрямна лінія – guiding line
напрямок – direction
натуральна величина – natural size
нахил – inclination
наявність – availability
невидимий – invisible
невласний центр – improper centre
незакономірна поверхня – accidental surface
незмінний – constant
нерозгортна поверхня – folded surface
нерухома пряма – fixed straight line
нерухомий – fixed
нижній – lower
нуль – zero

О

обертання – rotation
образ – image
обрис – outline
однополосний гіперболоїд обертання – hyperboloid of rotation of one sheet
опорна точка – anchor point
опускати – put down
ордината – ordinate
основа – base
отримана точка – obtained point

П

парабола – parabola
параболоїд – paraboloid
паралелограм – parallelogram

паралель – parallel
паралельний – parallel
паралельність – parallel position
первісне положення – initial position
передній – front
переміщення – movement
переміщення безперервне поступальне – continuous progressive movement
переміщення гвинтове – helix movement
переміщувати – move
переміщуватись по колу – move in a circle
переріз – section
перетин – intersection
перетворення (креслення) – conversion
перетворення (чого-небудь у що-небудь) – transformation
перпендикуляр – perpendicular
перпендикулярний – perpendicular
перпендикулярність – perpendicularity
перпендикулярно – athwart
півсфера – hemisphere
піраміда – pyramid
плавна крива лінія – smooth curve
плоске зображення – plane image
плоскопаралельне переміщення – planar movement
площина – plane
площина паралелізму – plane of parallelism
площина проекцій – projection plane
побудова – construction
побудувати – construct
повертати за годинниковою стрілкою – rotate clockwise
повертати проти годинникової стрілки – rotate counterclockwise
поверхня – surface
поверхня обертання – surface of rotation
поверхня переносу – moving surface
позитивне значення – positive value
позиційний – positional
позначати – mark
позначка – mark
попарно – in pairs
послідовність – succession
постійний радіус – constant radius
початок координат – origin
призма – prism
продовження – extension

проектуватись – be projected
проекціювання – projection
проекціююча пряма – projecting straight line
проекціюючий промінь – projecting ray
промінь – ray
простір – space
просторовий – spatial
профільна пряма – profile straight line
пряма – straight line
пряма рівня – level straight line
прямий круговий конус – right circular cone
прямокутний – right-angled
прямокутний трикутник – right-angled triangle
прямокутник – rectangle
прямолінійчастий – rectilinear

Р

радіус – radius
ребро – edge
ребро звороту – curved edge
рівномірно – uniformly
розвертати, обертати – rotate
розв'язання – solving
розгортка – evolvent
розгортна поверхня – unfolded surface
розкатка – unfolding
розміщення – location

С

сітка поверхні переносу – mesh of moving surface
січний – intersecting
складка – fold
слід – trace
співосний – coaxial
співпадати – coincide
спільне дотичне коло – common tangent circle
спільний – common
спіраль – spiral
спотворення – deformation
стрілка – arrow
суміжний – adjacent
суміщення – superposition
сусіднє ребро – adjacent edge

сусідня сторона – adjacent side
сфера – sphere
сходитись – converge

Т

твірна лінія – generating line
твірне коло – generating circle
теорема – theorem
тор – torus
точка – point
траєкторія руху – trajectory of motion
трикутна піраміда – triangular pyramid
трикутна призма – triangular prism
трикутник – triangle
трубчаста поверхня – tubular surface

У

Ф

фігура – figure
форма – form
фронталь – front straight line

Х

Ц

циклічна поверхня – circular surface
циліндр – cylinder
циліндрична поверхня – cylindrical surface
циліндроїд – cylindroid
циркуль – compasses

Ч

чверть – quarter
чотирикутна призма – four-angled prism
чотирикутник – quadrangle

Ш

штрихова лінія – hatching line

Щ

Ю

Я